Developing Mathematical Thinking Institute

The Developing Mathematical Thinking Institute (DMTI) is dedicated to enhancing students' learning of mathematics by supporting educators in the implementation of research-based instructional strategies through high-quality professional development. For more information contact Dr. Brendefur at brendefur.dmti@gmail.com

Developing Mathematical Thinking: Achieving Success in Title I Schools

NATIONAL TITLE I CONFERENCE

LONG BEACH, CA

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Session Overview

- What does it mean to Develop Mathematical Thinking?
  - DMT Framework
- DMT Examples
  - Fact Fluency
  - Place Value
  - Fractions
- Evidence to Support DMT
- DMTI Professional Development and Resources

What does it mean to Develop Mathematical Thinking?

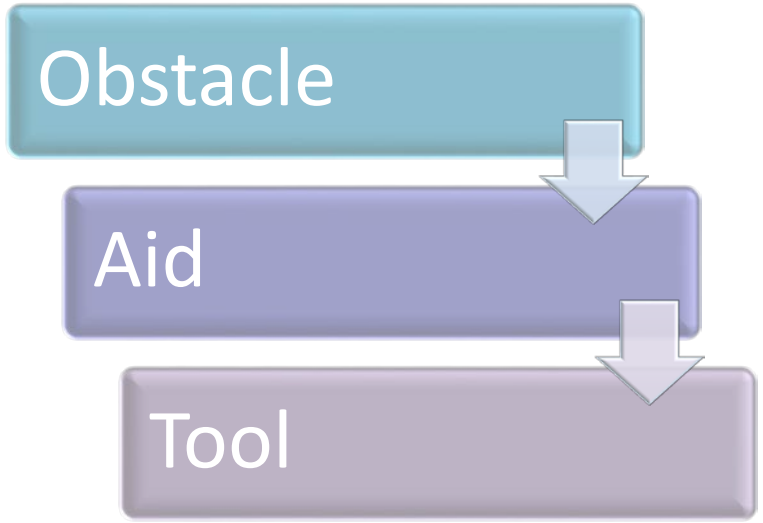
DMT is built on a theoretical foundation drawing from three major learning theories:

Cognitive Theories

Social Interactional Theories

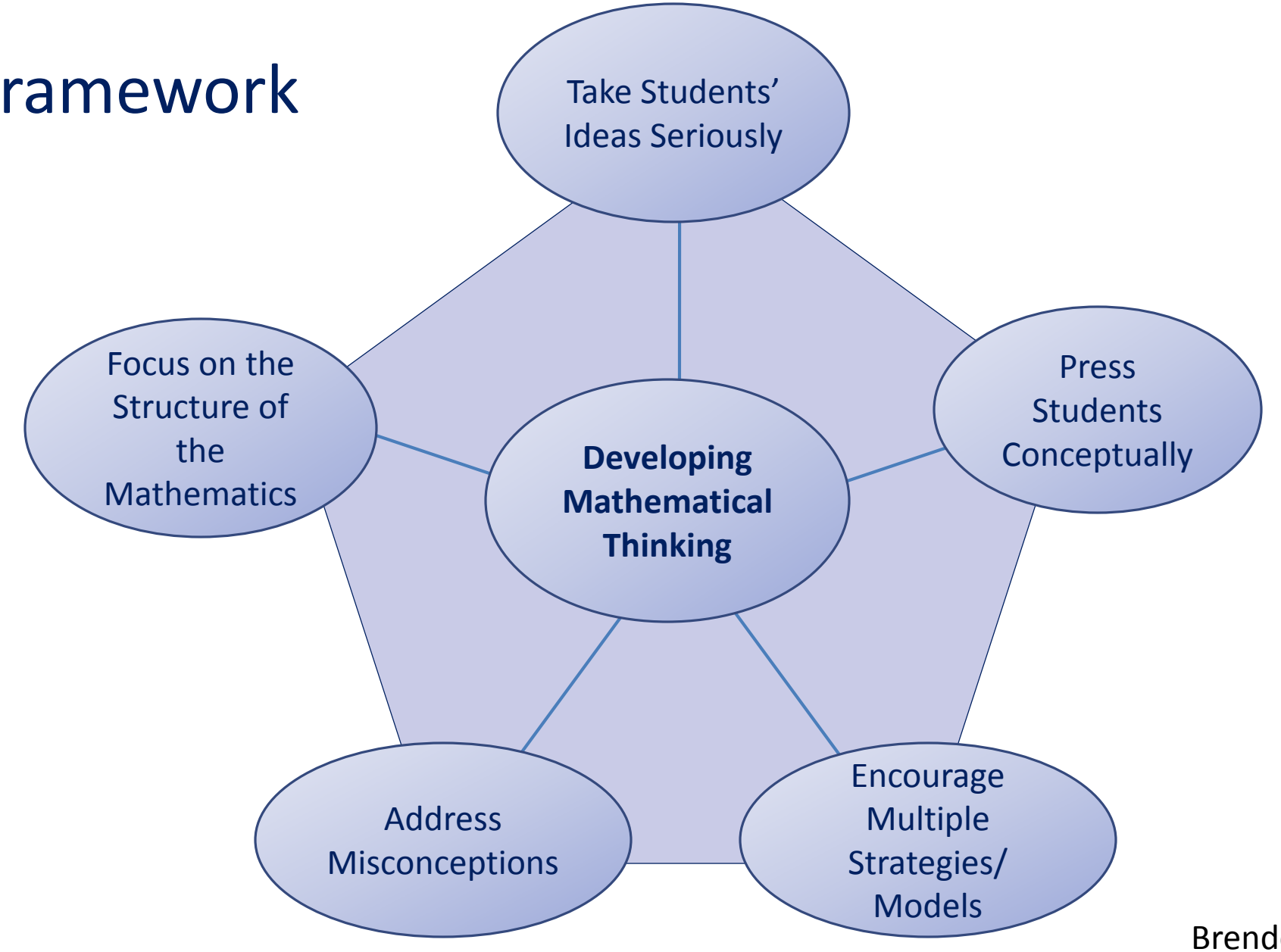
Behaviorism

DMT as a process…



DMT as a structure…

DMT Framework



Brendefur et al., 2008

DMT in Schools

Professional Development Courses and Workshops

Unit Studies

In-Class Support

Resources

- Focusing Calendars
- Unit Overviews
- Curricular Modules
- Common Assessments
- Primary Mathematics Assessment: Screener and Diagnostic

Fact Fluency

Fact Fluency Example: Grade 3

Solve 12 x 13 in your head. How did you find the product?

How could you solve 6 x 8 if you didn't know the fact or you forgot it?

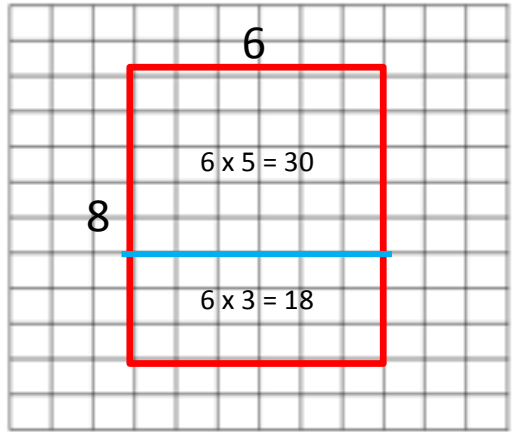
Strategy Card Template

1	2	3	4	5	6
8	16	24	32	40	48

6x8

Front

(6 x 5) + (6 x 3) = 30 + 18 = 6 x 8 = 48



6 x 5 = 30
6 x 3 = 18
6 x 8 = 48

Back

I know *6 x 5 = 30.*
I also know *6 x 3 = 18.*
30 + 18 = 48
So that means *6 x 8 = 48.*

Place Value Example

GRADE 1

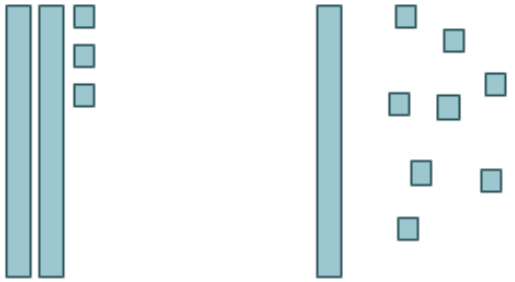
Word Bank	
Unit size	Decompose
Compose	
Partition	Iterate

Place Value Example: Grade 1

A key conceptual understanding in Grade 1 is the place value composition of two-digit numbers.

Students in Grade 1 must understand how units of ten and one can be used to compose/decompose numbers in flexible ways.

To address this, research suggests a linear model (or bar model) can support students' proportional reasoning and number sense more than more traditional set models (e.g. base ten pieces)



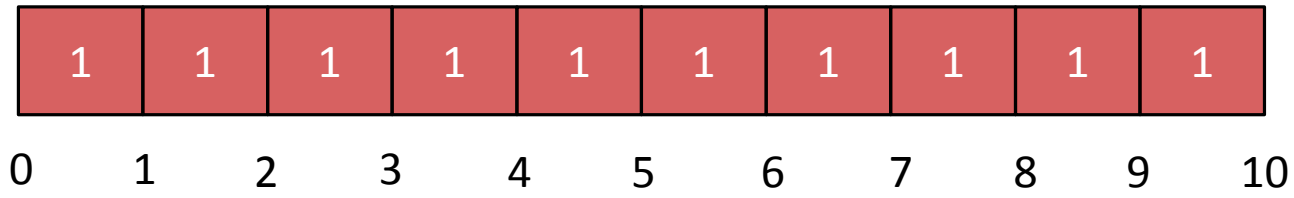
"18 is greater than 23?"

Battista & Clements, 2000; Brendefur & Strother, 2016

Word Bank	
Unit size	Decompose
Compose	
Partition	Iterate

Lesson 6: Units of 1 and Units of 10

1. Use your unit cube to build a bar model for 10 by ***iterating*** units of one.



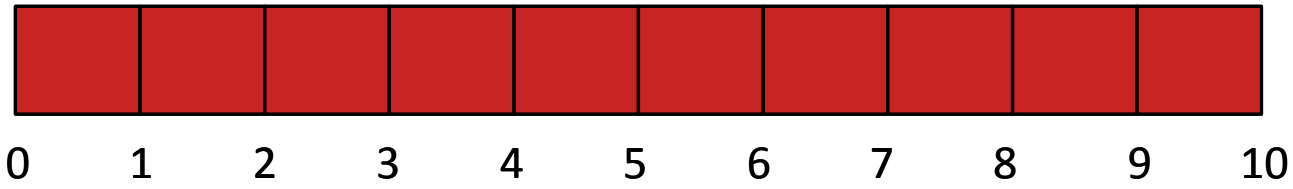
Materials needed: *Blank 12x18 paper (turned to a landscape orientation) and a single cube for each student.*

Word Bank	
Unit size	Decompose
Compose	
Partition	Iterate

Lesson 6: Units of 1 and Units of 10

2. Now build another bar model that is a unit of 10 as shown below.
3. How would you describe what is the same and what is different about these two bar models? Discuss this with a partner.

"This bar model is 10 units of one. It is the same size as the 1 unit of ten but we are counting in different units."



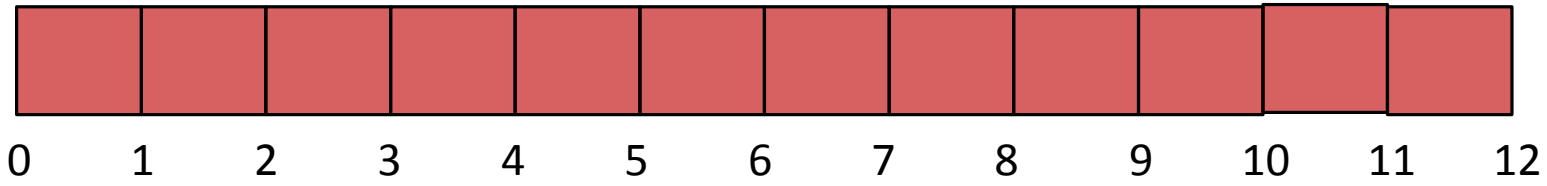
"This bar model is 1 unit of ten. It is the same size as the 10 units of one but we are counting in different units. "

Word Bank	
Unit size	Decompose
Compose	
Partition	Iterate

Lesson 6: Units of 1 and Units of 10

4. Iterate new units on your bar models of 10 to make them bar models of 12.
5. Describe the units that are being counted to make 12 and how they are different in each of the bar models.

"12 units of one."



"1 unit of ten and 2 units of one."

Lesson 6: Units of 1 and Units of 10

Word Bank	
Unit size	Decompose
Compose	
Partition	Iterate

a. Draw two different bar models for each of the following numbers.

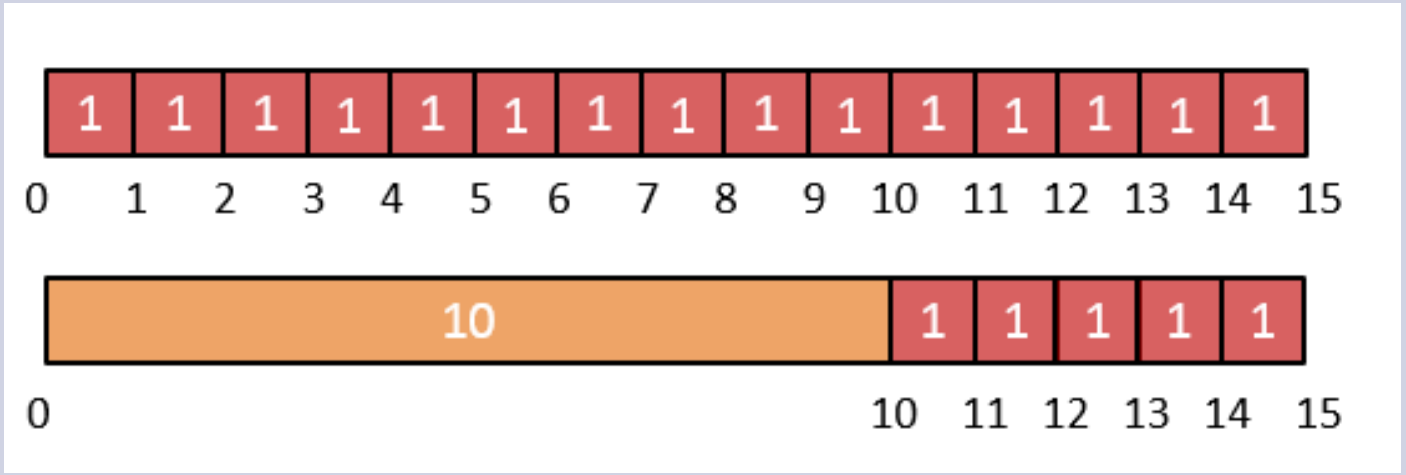
b. Use only units of 1 for your first bar model.

c. Use a combination of units of 1 and units of 10 in your second bar model.

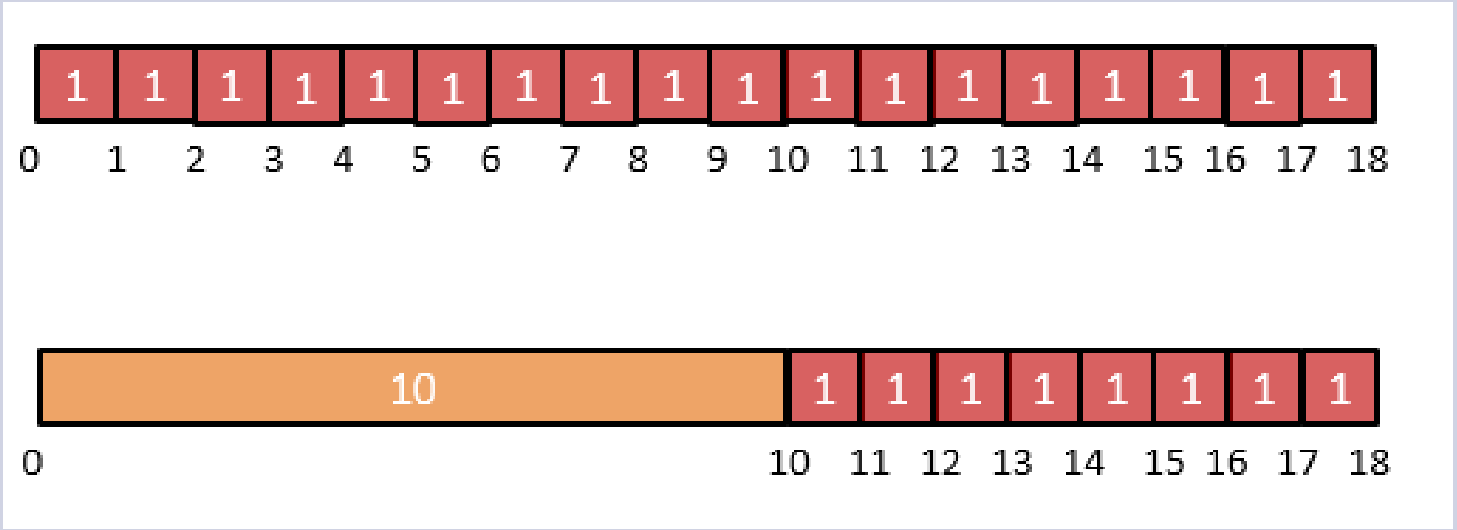
d. Talk about the different units in your bar models with a partner.

Number
15
18
20

Lesson 6: Units of 1 and Units of 10

Number	*Bar Model Examples*
15	 *"15 units of one."* *"1 unit of ten and 5 units of one."*

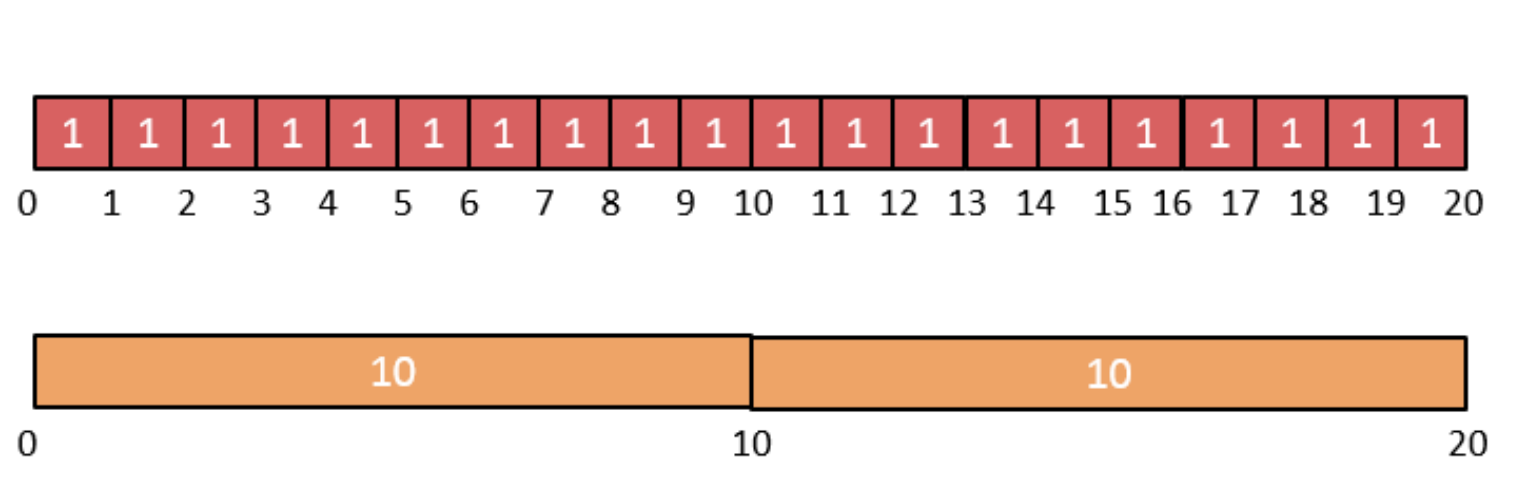
Lesson 6: Units of 1 and Units of 10

Number	*Bar Model Examples*
18	

"18 units of one."

"1 unit of ten and 8 units of one."

Lesson 6: Units of 1 and Units of 10

Number	*Bar Model Examples*
20	

"20 units of one."

"2 units of ten."

Fraction Example

GRADES 4 AND 5

Rethinking Fractions

$$\frac{\text{numerator}}{\text{denominator}} \qquad \frac{\text{part}}{\text{whole}}$$

Rethinking Fractions

$$\frac{\text{numerator}}{\text{denominator}} \qquad \frac{\text{count}}{\text{unit size}} \qquad \frac{\text{iterations}}{\text{partitions}}$$

(units to compose 1)

Lesson 3: Fractions

Word Bank	
Unit size	Iterate
Partition	

Draw a ***model*** that matches the following problem:

You have a ribbon that is 4 feet in length. If you cut a piece off that is $\frac{1}{3}$ of the whole ribbon, how long (in feet) would the piece you cut be?

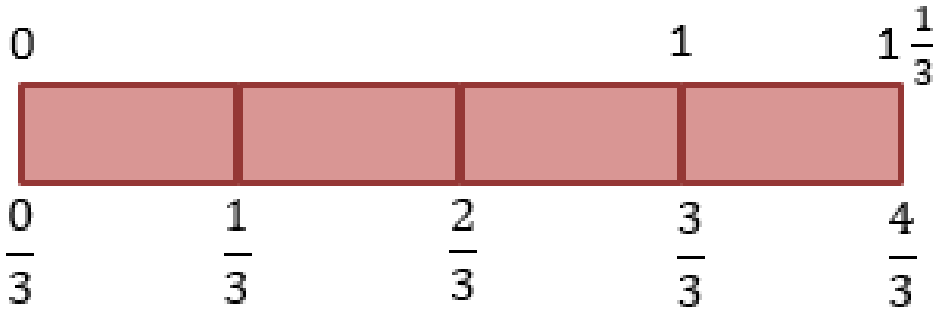
Lesson 3: Fractions

Word Bank	
Unit size	Iterate
Partition	

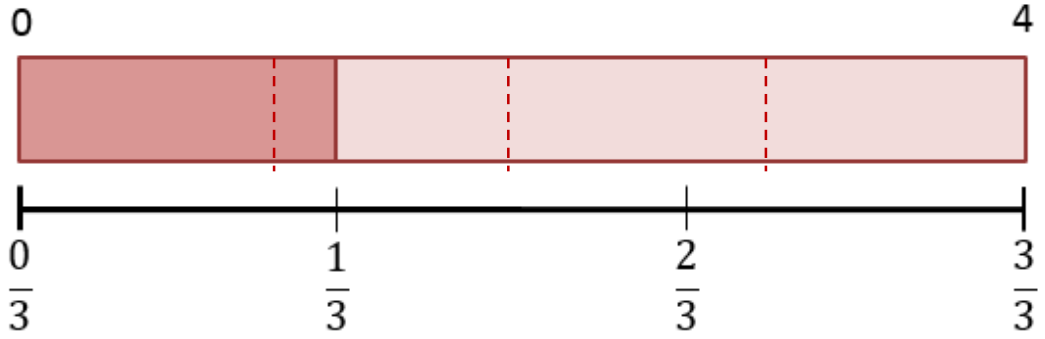
You have a ribbon that is 4 feet in length. If you cut a piece off that is $\frac{1}{3}$ of the whole ribbon, how long (in feet) would the piece you cut be?

Which model best represents the problem? Why?

Model A



Model B



Lesson 3: Fractions

Word Bank
Unit size Iterate Partition

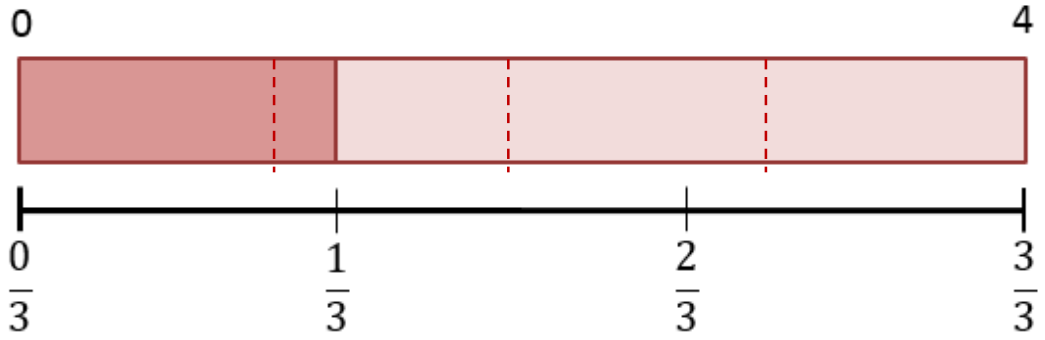
You have a ribbon that is 4 feet in length. If you cut a piece off that is $\frac{1}{3}$ of the whole ribbon, how long (in feet) would the piece you cut be?

Which model best represents the problem? Why?

*Model B best represents the problem because it shows that we are **partitioning** a whole amount of 4 into thirds.*

$$\frac{1}{3} \times 4$$

Model B



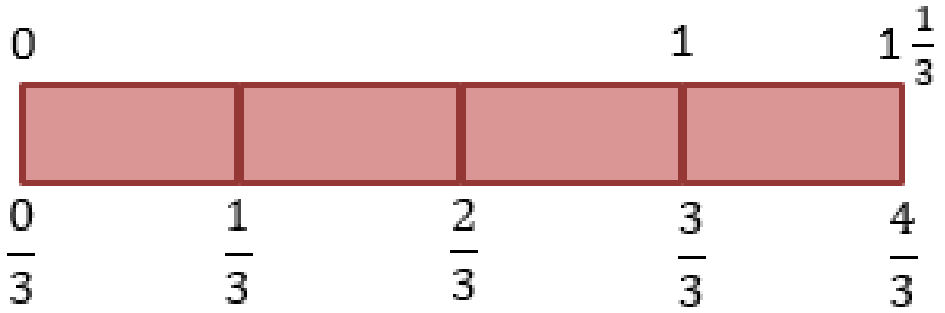
Lesson 3: Fractions

Word Bank
Unit size Iterate Partition

You have a ribbon that is 4 feet in length. If you cut a piece off that is $\frac{1}{3}$ of the whole ribbon, how long (in feet) would the piece you cut be?

Why is Model A not the best way to represent the problem?

Model A



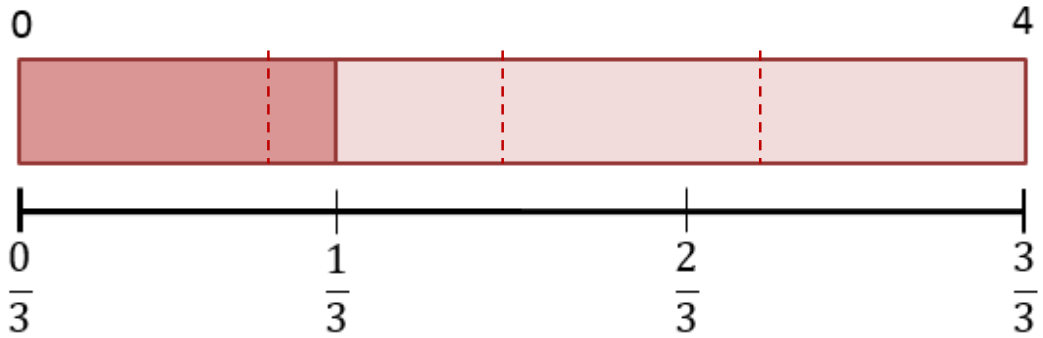
*Model A does not correctly model the problem because the problem is not telling us to **iterate** a unit of $\frac{1}{3}$ four times.*

Lesson 3: Fractions

Word Bank
Unit size Iterate Partition

Now let's examine how to model problems that involve multiplying a whole number by a fraction in the form $\frac{a}{b}$ x n.

Thinking back to the previous problem, let's describe what is challenging about finding the product of $\frac{1}{3}$ x 4.



Because we are not iterating by a whole number, it is difficult to name $\frac{1}{3}$ of 4 precisely.

Lesson 3: Fractions

Word Bank	
Unit size	Iterate
Partition	

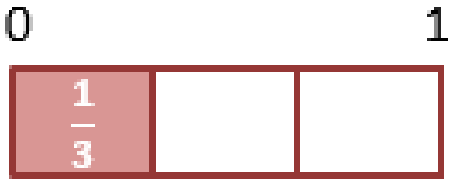
Let's think about a related, but easier problem.

Instead of $\frac{1}{3}$ x 4 let's change the problem to $\frac{1}{3}$ x $1 = \frac{1}{3}$

Now, let's change from 1 unit of one to 2 units of one. $\left(\frac{1}{3} \text{ x } 1\right) \text{x } 2 = \frac{2}{3}$

Change the problem to this. $\left(\frac{1}{3} \text{ x } 1\right) \text{x } 3 = \frac{3}{3}$

And finally, change the problem to this. $\left(\frac{1}{3} \text{ x } 1\right) \text{x } 4 = \frac{4}{3}$

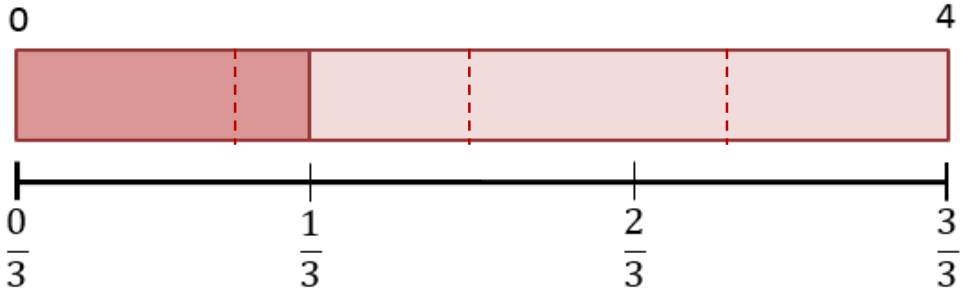


Lesson 3: Fractions

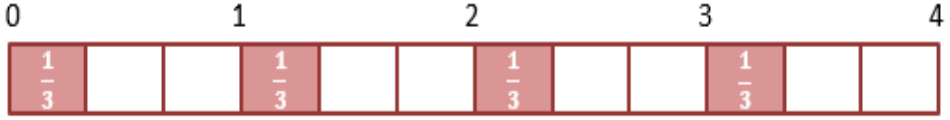
Word Bank
Unit size Iterate Partition

To connect the models we have just drawn to our original model of the problem $\frac{1}{3}$ x 4 = $\frac{4}{3}$ = 1 $\frac{1}{3}$, let's create a final drawing that shows all of the $\frac{1}{3}$ units placed together to show what $\frac{1}{3}$ of 4 is.

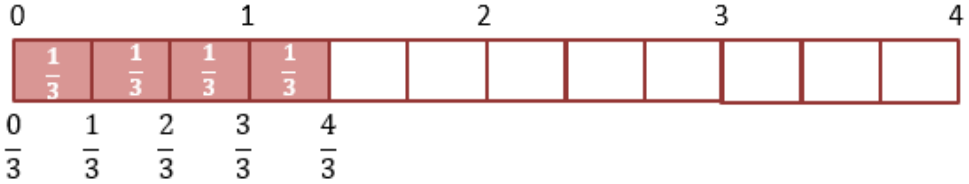
Our original model



This model shows all of the $\frac{1}{3}$ units in ($\frac{1}{3}$ x 1) x 4.



This model that shows all of the $\frac{1}{3}$ units in ($\frac{1}{3}$ x 1) x 4 put together the explain why $\frac{1}{3}$ x 4 = $\frac{4}{3}$ = 1 $\frac{1}{3}$



Lesson 3: Fractions

Word Bank
Unit size Iterate Partition

Now, use what we have learned about modeling multiplication the form $\frac{a}{b} \times n$ to model and solve this version of the earlier problem.

You have a ribbon that is 3 feet in length. If you cut a piece off that is $\frac{1}{4}$ of the whole ribbon, how long (in feet) would the piece you cut be?

A. Draw a model to represent the problem.

B. Write an equation or expression that matches the problem and your model.

C. Use several models to show how you can solve the problem. Make sure to think about the problem in the way it might relate to $\frac{1}{4} \times 1$.

Name:_____

Expression	Word Problem	Model(s)	Solve
$\frac{1}{2}$ x 3			
$\frac{1}{3}$ x 4			
$\frac{1}{5}$ x 4			
$\frac{1}{4}$ x 5			

Summary of DMT Examples

Conceptually understanding the topic and connecting it to the procedures.

Use of enactive, iconic, and symbolic models.

Focus on the structural components (language of units, decomposing, composing, partitioning, and iterating).

Evidence to Support DMT

TECHNICAL REPORTS, PUBLISHED RESEARCH AND EXTERNAL EVALUATIONS

Developing Mathematical Thinking (MSP Grant)

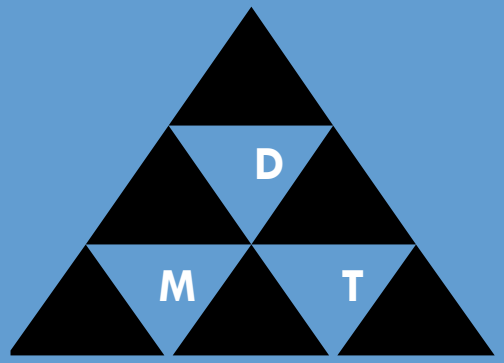
Years: 2004 – 2007

Participants: 3 Title I Schools in rural, suburban, and suburban locations.; Grades K - 6

Treatment: DMT professional development (5 days summer professional development followed by 8 days of embedded professional development)

Instrument: ISAT achievement instrument (Grades 3 – 6)

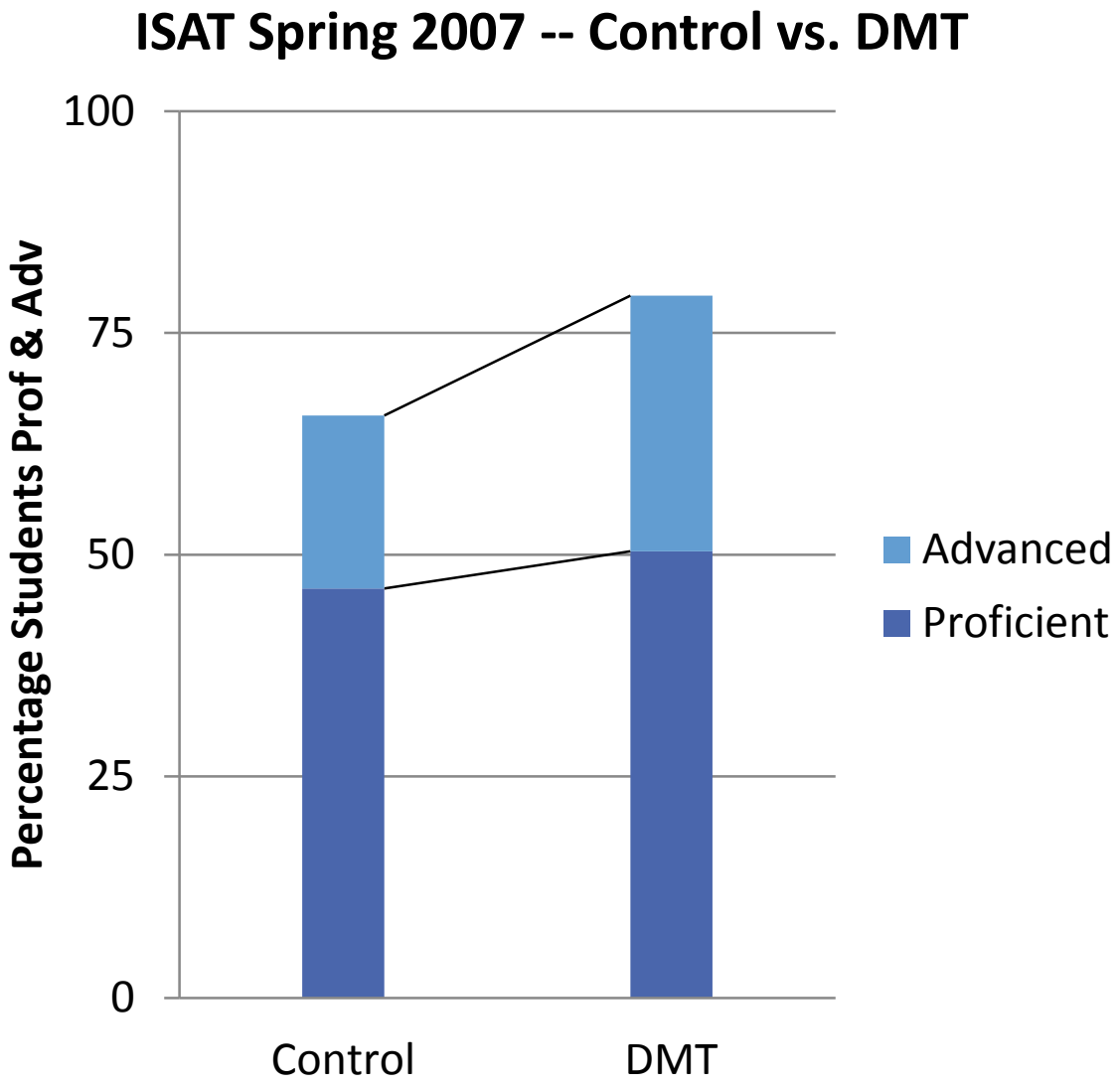
Timeline: 3 years



79.2% of students in DMT teachers' classrooms scored proficient or advanced compared to 65.7% in comparison teachers' classrooms

- These differences are statistically significant (z=2.603, p < .01)

RMC (2008)



Developing Mathematical Thinking 2 (MSP Grant)

Years: 2007 – 2010

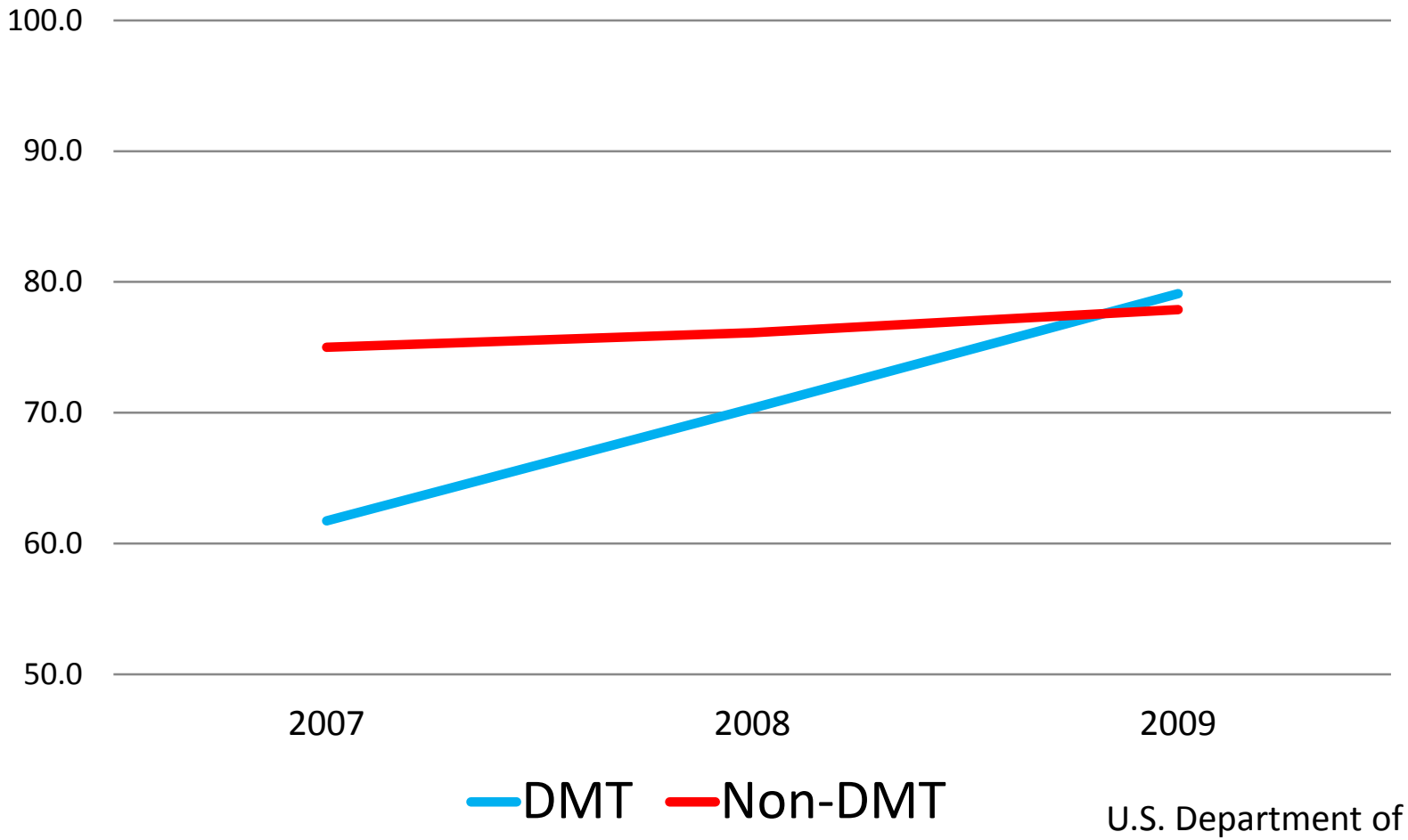
Participants: 3 elementary Title I schools were chosen randomly out of 6 and 1 out of 2 middle schools; all schools had between 80 and 95% SES and over 60% ELL and migrant population. Grades 3 - 8

Treatment: DMT professional development (5 days summer professional development followed by 8 days of embedded professional development)

Instrument: ISAT achievement instrument (Grades 3 – 8)

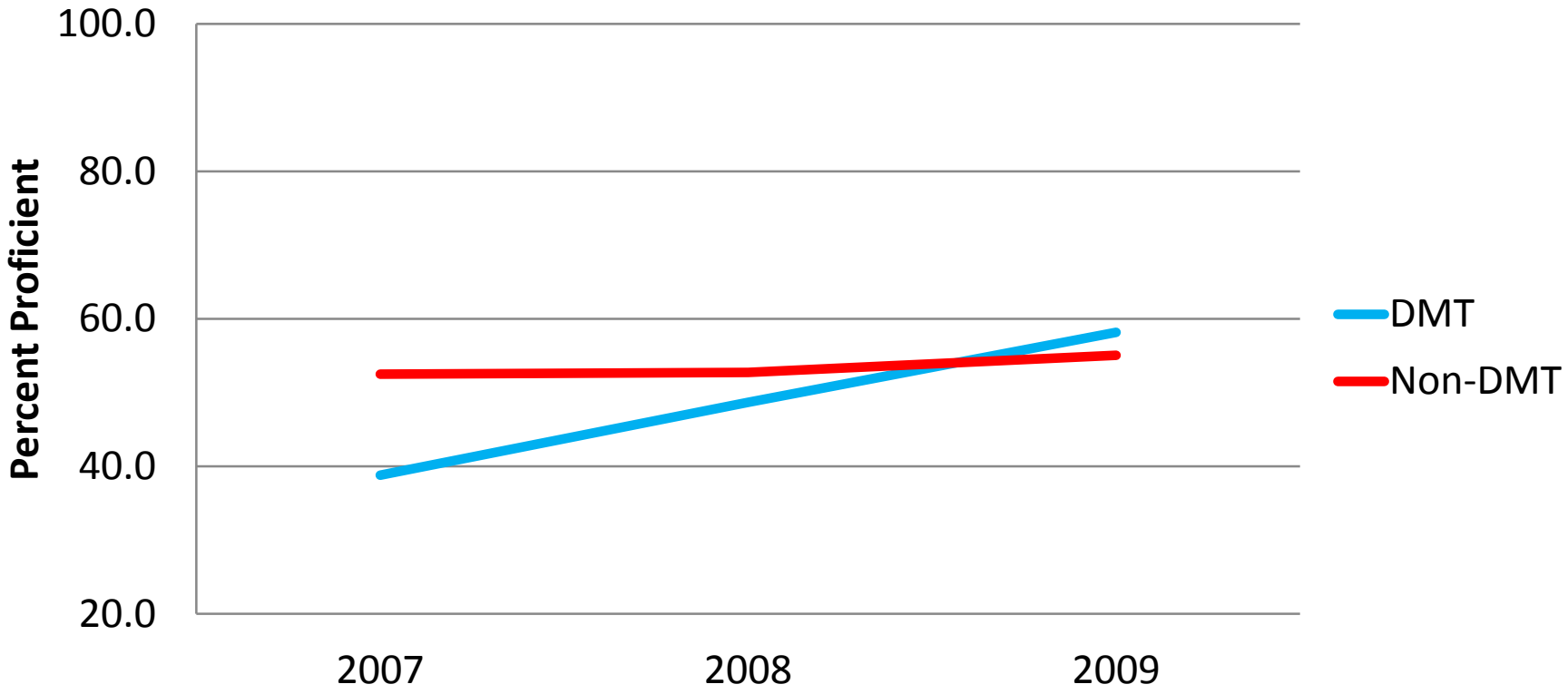
Timeline: 3 years

Overall School Proficiency



U.S. Department of Education, 2010

Schools ELL Proficiency: Grades 3 - 8



U.S. Department of Education, 2010

Improving Teacher Monitoring of Learning (IES Grant)

Participants: 8 Title I Schools randomly chosen into Control and Treatment groups; Grades K - 5

Treatment: DMT professional development (3 days summer professional development followed by 12 days of embedded professional development)

Control: Formative assessment professional development (3 days summer professional development followed by 12 of embedded professional development)

Instrument: MAP achievement instrument (Grades 2 – 5)

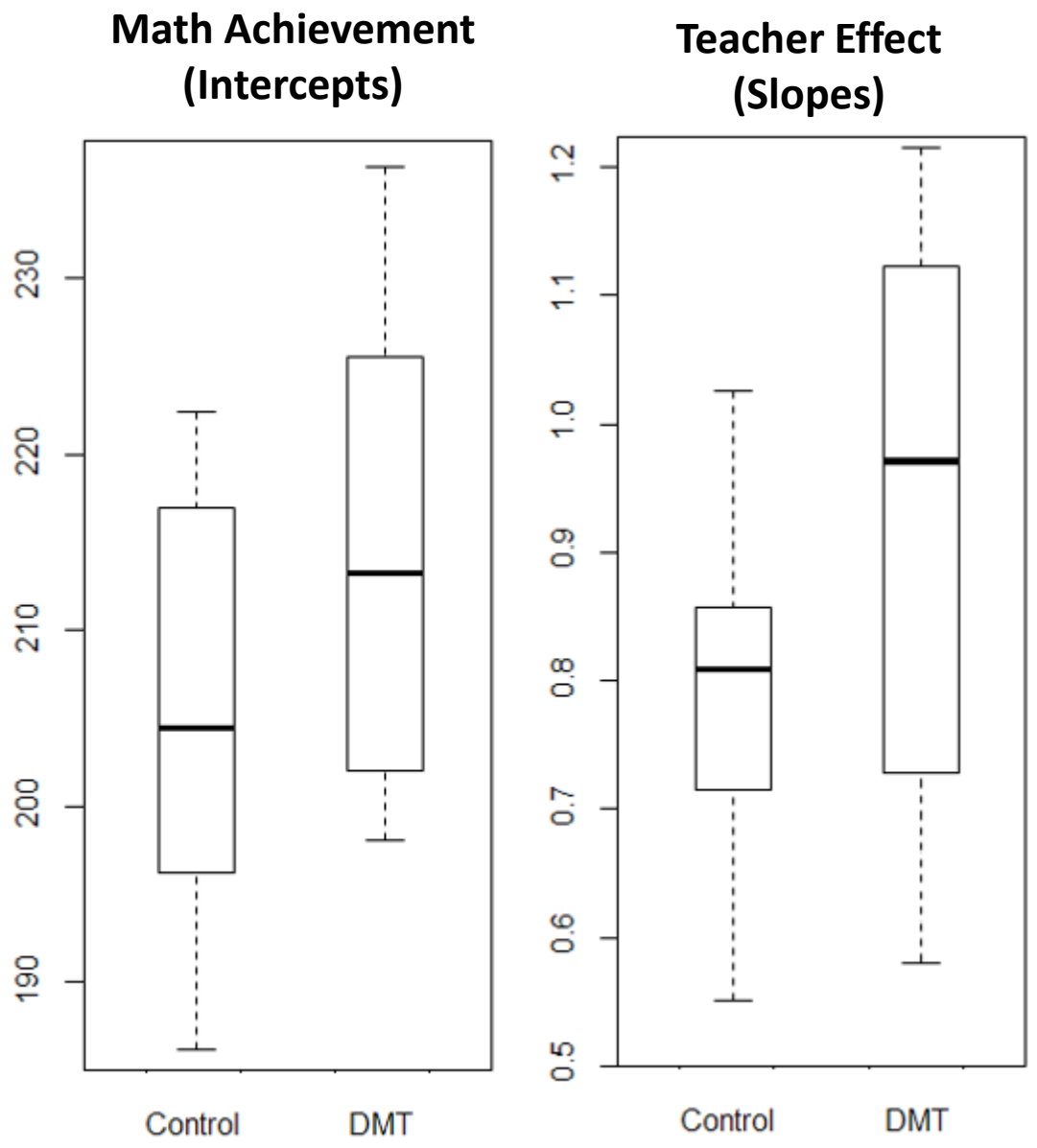
Timeline: 2 years

IES Grant

Results: using Hierarchal Linear Modeling (HLM)

- DMT had a significant positive effect on student achievement.

- The more teachers were observed implementing DMT in the treatment schools, the greater the student achievement.



Multiplication Fact Fluency

Participants: 6 schools – 3 title I and 3 non-title I; Grades 3 - 5

Treatment: 10 minutes a day of differentiated practice (visual arrays, decomposing, and language)

Control: 10 minutes a day of rote practice (worksheets, flash cards, games and other tasks)

Instrument: 2 minute multiplication probe

Timeline: 4 weeks

Multiplication Fact Fluency

Figure 1. Grade 3 performance on a multiplication fluency pretest and posttest by group.

Error bars represent the standard error of the mean.

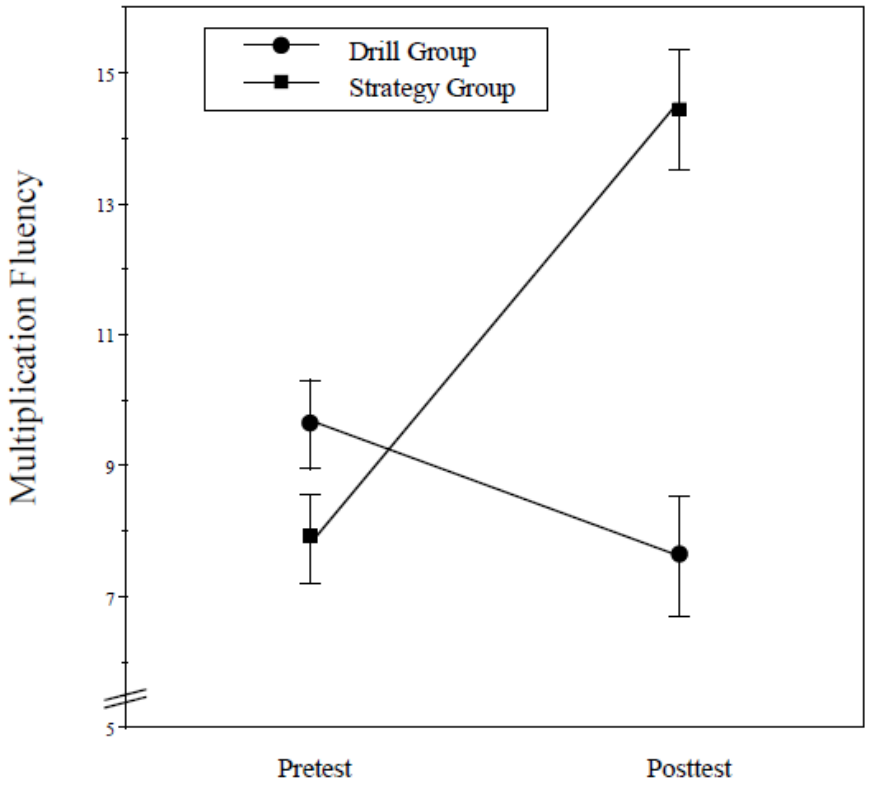
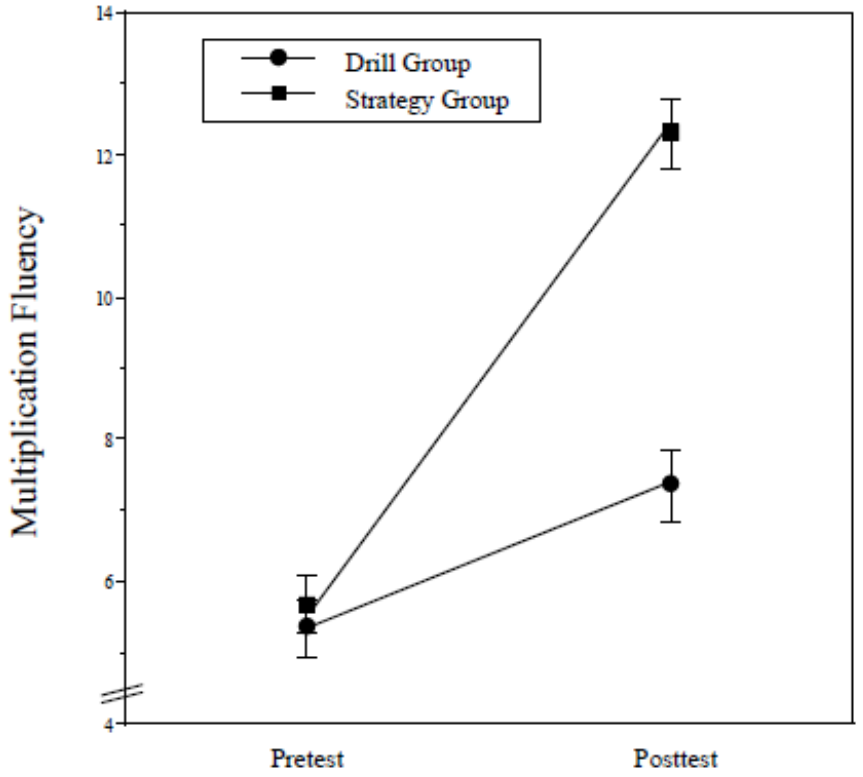


Figure 2. Grade 4 performance on a multiplication fluency pretest and posttest by group.

Error bars represent the standard error of the mean.



Brendefur, Strother, Thiede & Appleton, 2015

K-2 PMA Data

Kindergarten -- Whittier

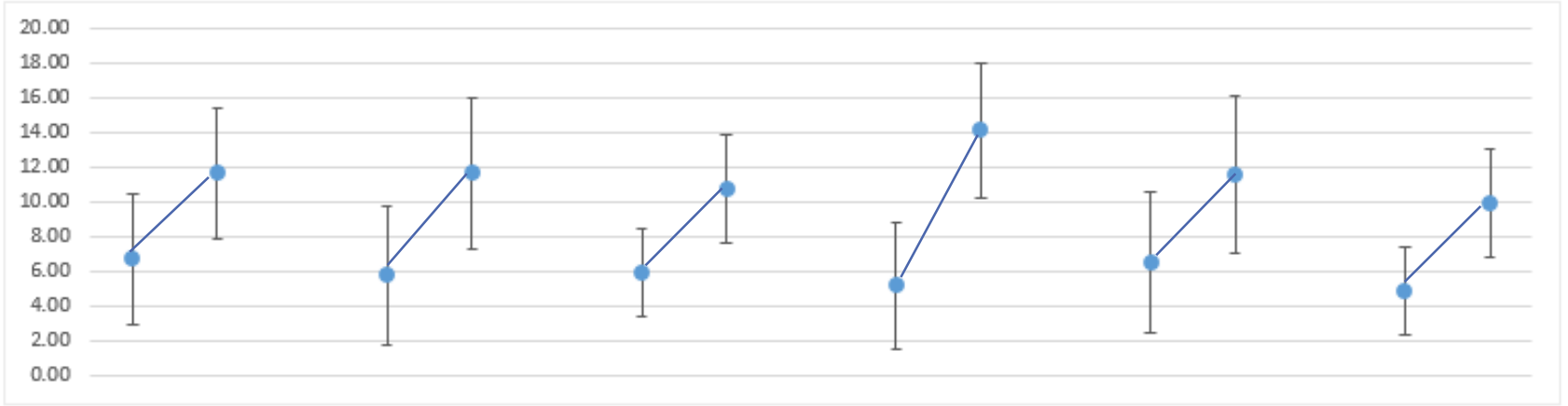
2014-2015

Boise School District

Whittier

Grade: K

	Sequencing		Facts		Relational Th		Context		Measurement		Space	
	Fall	Winter	Fall	Winter	Fall	Winter	Fall	Winter	Fall	Winter	Fall	Winter
Ave.	7.03	12.19	5.07	9.61	6.87	9.18	5.81	12.25	6.03	10.33	6.37	9.54
S.D.	3.13	3.63	3.60	4.24	2.59	2.83	3.72	4.18	3.91	4.40	2.62	3.06
Ave.	6.72	11.65	5.78	11.66	5.90	10.71	5.19	14.10	6.55	11.54	4.88	9.89
S.D.	3.75	3.76	4.00	4.33	2.53	3.12	3.68	3.87	4.03	4.54	2.49	3.13
BSD (gain)		*5.16*		*4.54*		*2.31*		*6.44*		*4.30*		*3.17*
Tchr (gain)		*4.93*		*5.88*		*4.81*		*8.91*		*4.99*		*5.01*



First Grade - Whittier

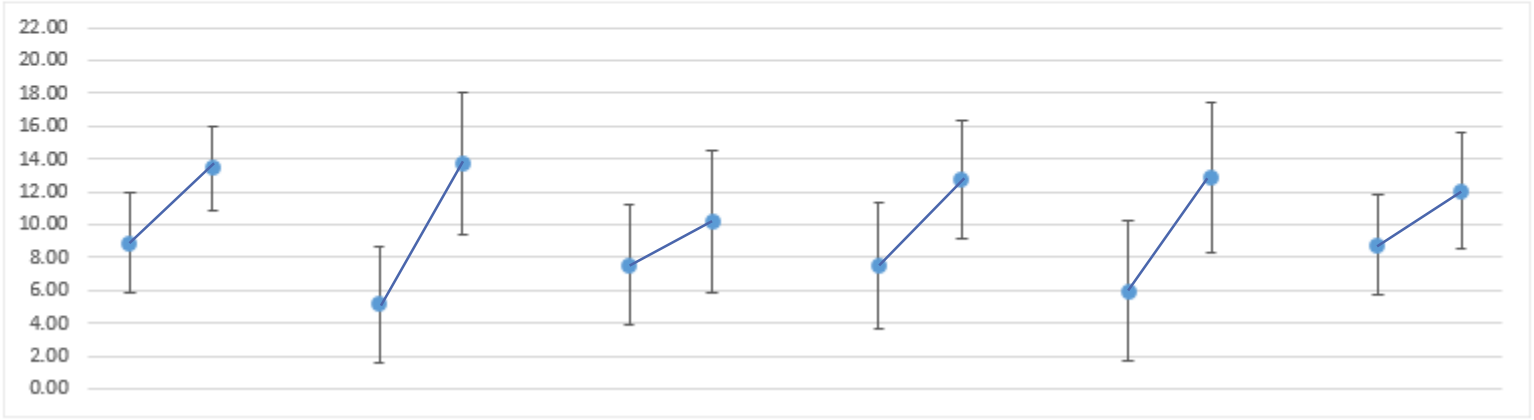
2014-2015

Boise School District

Whittier

Grade: 1

	Sequencing		Facts		Relational Th		Context		Measurement		Space	
	Fall	Winter	Fall	Winter	Fall	Winter	Fall	Winter	Fall	Winter	Fall	Winter
Ave.	11.55	15.20	7.93	15.07	7.69	12.02	10.56	15.63	10.07	14.28	10.80	14.22
S.D.	2.83	1.95	4.30	4.07	3.69	4.44	3.88	2.94	4.55	4.14	3.01	2.79
Ave.	8.86	13.43	5.14	13.72	7.54	10.19	7.48	12.73	5.96	12.87	8.75	12.06
S.D.	3.03	2.58	3.55	4.38	3.63	4.28	3.86	3.57	4.29	4.63	3.08	3.50
BSD (gain)		*3.65*		*7.14*		*4.33*		*5.07*		*4.21*		*3.42*
Tchr (gain)		*4.56*		*8.57*		*2.65*		*5.25*		*6.91*		*3.31*



Second Grade - Whittier

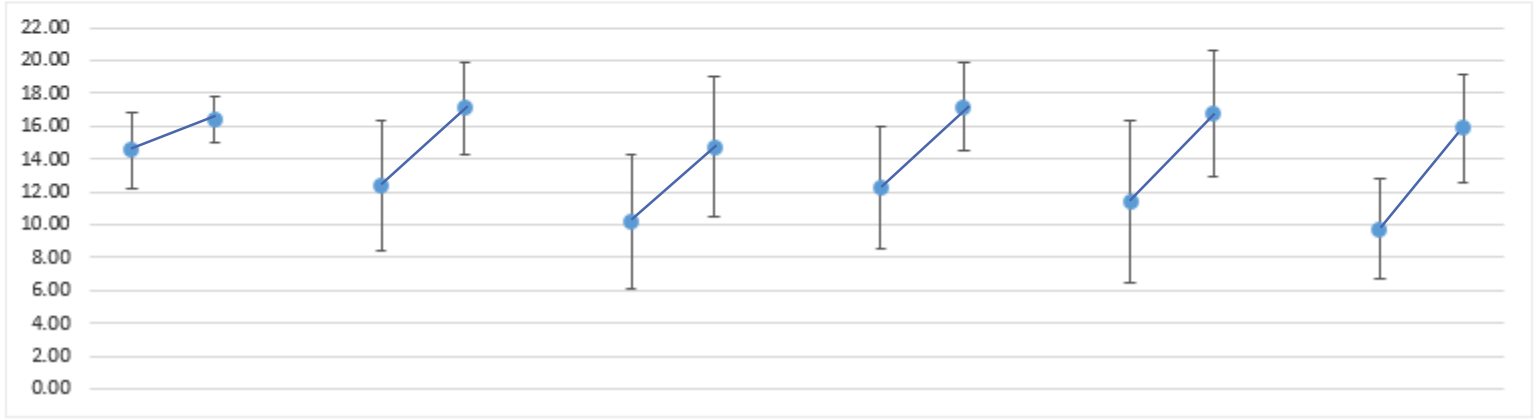
2014-2015

Boise School District

Whittier

Grade: 2

	Sequencing		Facts		Relational Th		Context		Measurement		Space	
	Fall	Winter	Fall	Winter	Fall	Winter	Fall	Winter	Fall	Winter	Fall	Winter
Ave.	15.00	16.05	14.58	17.18	12.87	15.26	14.60	16.66	13.52	16.49	12.64	15.39
S.D.	2.14	1.50	3.54	2.60	4.11	3.93	3.27	2.64	4.53	3.87	3.21	2.92
Ave.	14.52	16.45	12.39	17.10	10.17	14.71	12.23	17.17	11.45	16.76	9.74	15.87
S.D.	2.28	1.41	4.00	2.83	4.08	4.27	3.74	2.70	4.94	3.85	3.05	3.27
BSD (gain)		*1.05*		*2.60*		*2.39*		*2.06*		*2.97*		*2.75*
Tchr (gain)		*1.93*		*4.71*		*4.54*		*4.94*		*5.31*		*6.14*



Developing Mathematical Thinking Institute

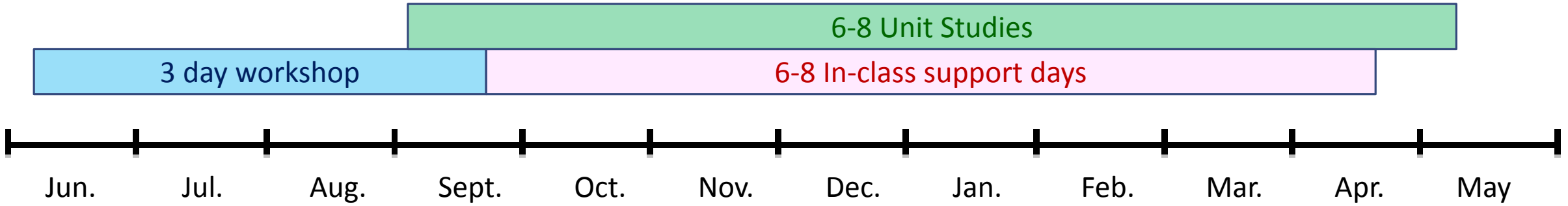
PROFESSIONAL DEVELOPMENT AND CURRICULAR RESOURCES

Professional Development Plan

Professional Development	*DEVELOPING MATHEMATICAL THINKING*		
	Year 1	**Year 2**	**Year 3**
Focus Area	*Number & Algebra*	*Measurement & Geometry*	*Probability & Statistics*
Summer PD 5 Days (45 hours) In-depth topics	√	√	√
Ongoing PD 18 days Unit Study (**4 X semester**) Observations (**monthly**) Demonstrations (**monthly**)	√	√	√

Current Professional Development Plan: 2 to 3 Year Partnership

- 3 day professional development workshop before school begins or early in the school year
- 6-8 Unit Studies throughout the school year (on-site)
- 6-8 In-class support (team teaching, observations, model lessons)
- Curricular Resources



DMTI Curricular Resources

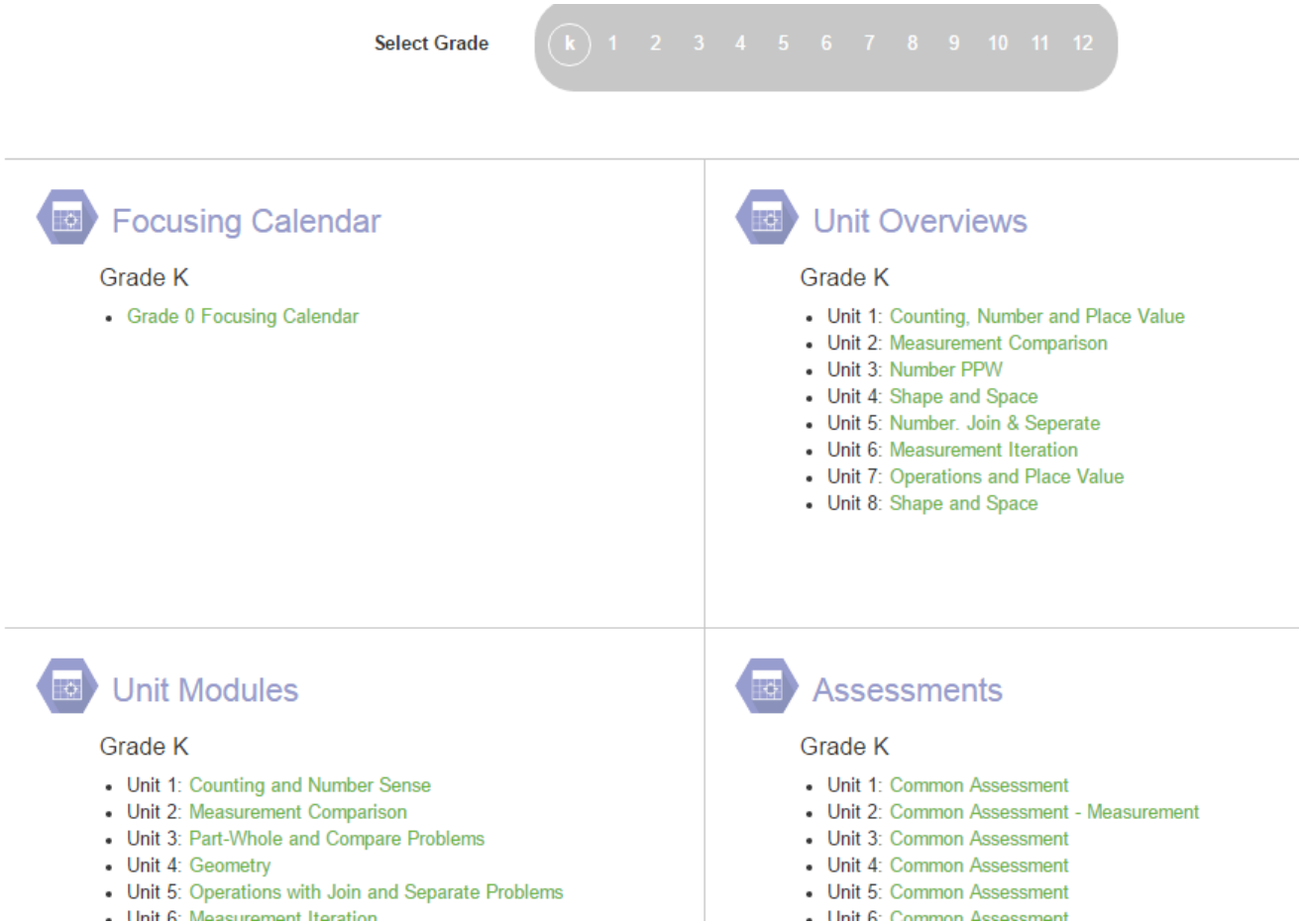
FOCUS CALENDARS, UNIT OVERVIEWS, MODULES AND ASSESSMENTS

Central Access Point

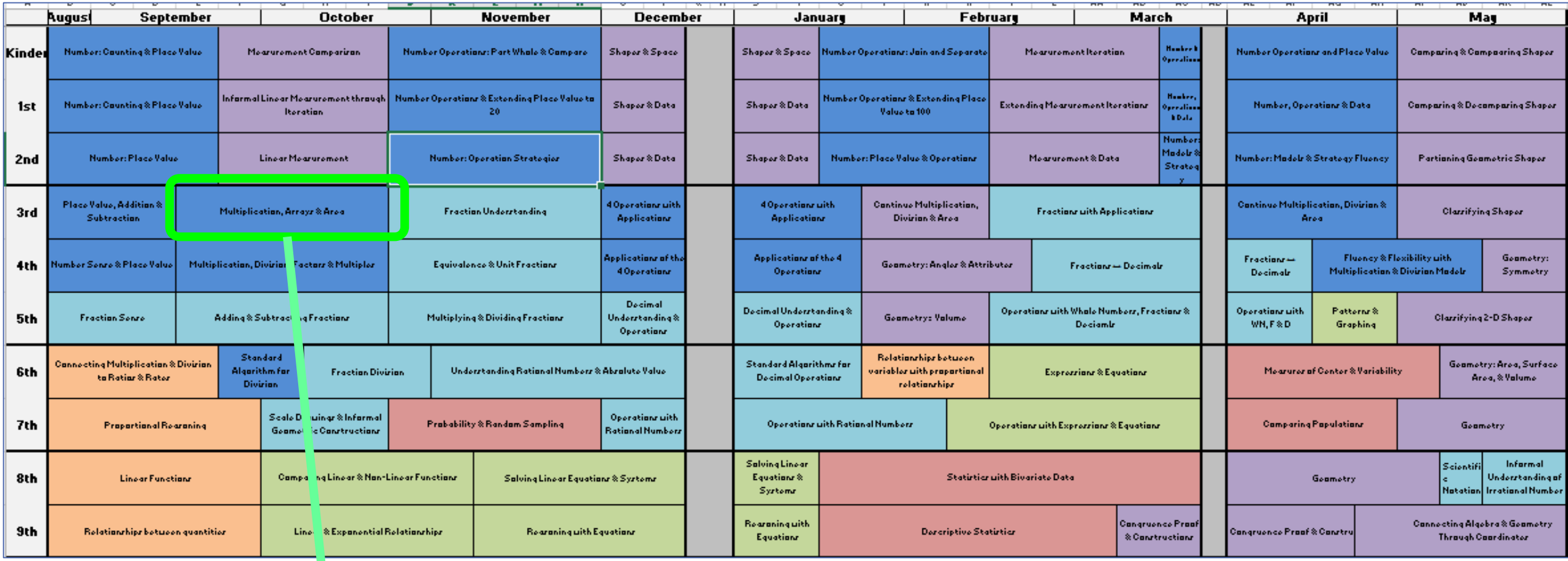
www.dmtinstitute.com

After logging in, users are given access to all available materials regardless of grade level.

This supports intervention and extension activities and informs teachers about the progression of content across grades.



K – 9 Curriculum Map



Each unit has an aligned **Unit Overview**, **Module** and **Common Assessment**

Grade 3 Common Assessment

Assessment: Problem Solving

Unit 2: Common Assessment (Grade 3) Name: Date:

1. There are 6 rows of desks in the third grade classroom. Each row has 4 desks in it. How many desks are there in the classroom?

Mary can grab 7 marbles (1 handful) out of the marble jar at a time. If she grabs 8 handfuls, how many marbles does she have?

2. Mary can grab 7 marbles (1 handful) out of the marble jar at a time. If she grabs 8 handfuls, how many marbles does she have? A ratio table has already been started for you. Complete the missing values.

Handful	1		3		5	6		8
Marbles	7	14					49	

3. Draw a double bar model or (a double number line) to match the ratio table above.

Assessment: Conceptual (Iconic model)

Unit 2: Common Assessment (Grade 3) Name: Date:

1. There are 6 rows of desks in the third grade classroom. Each row has 4 desks in it. How many desks are there in the classroom?

Draw a double bar model or (a double number line) to match the ratio table above.

2. Mary can grab 7 marbles (1 handful) out of the marble jar at a time. If she grabs 8 handfuls, how many marbles does she have? A ratio table has already been started for you. Complete the missing values.

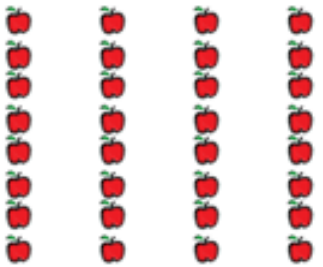
Handful	1		3		5	6		8
Marbles	7	14					49	

3. Draw a double bar model or (a double number line) to match the ratio table above.

Assessment: Skill and Conceptual

4. Carter has 32 jellybeans. If he shares them equally amo
receive?

5. Using the array to the right to complete the following:
   a. Write a multiplication equation that matches the array.
   b. Write a division equation that matches the array.
   c. Write a story problem that matches the array.



Using the array to the right to complete the following:

- Write a multiplication equation that matches the array.
- Write a division equation that matches the array.
- Write a story problem that matches the array.

Assessment: Justification and Reasoning

6. Use **4 different strategies** to represent the product for 4 x 9:

Repeated Addition (Iterate)	
Equal Groups	

Demi said that 3 x 4 and 4 x 3 are exactly the same. Explain using words and a drawing whether she is correct or not.

7. Demi said that 3 x 4 and 4 x 3 are exactly the same. Explain using words and a drawing whether she is correct or not.

DMTI Modules

Constructed in presentation software

Include printable, editable, worksheets as part of the lesson sequence

Successfully used as core or supplemental curricular resources

Currently, the entire school year (8-9 modules) are completed for grades K-6. (Grades 7 and 8 are underway.)

Grade 2

UNIT 3

NUMBER: *PLACE VALUE WITH PART WHOLE AND COMPARE PROBLEMS*

4-5 WEEKS

Module Sequence

Lesson 1: Counting Forward and Back

Lesson 2: Part-Whole Situations

Lesson 3: Part-Whole: Practice

Lesson 4: Part-Whole: Writing Contexts

Lesson 5: Part-Whole: Iconic Models

Lesson 6: Solving Compare Situations: Context

Lesson 7: Solving Compare Situations: Practice

Lesson 8: Solving Compare Situations: Iconic Models

Lesson 9: Solving Compare Situations: Symbolic Models

Lesson 10: Solving Compare Situations: Making Models and Justification

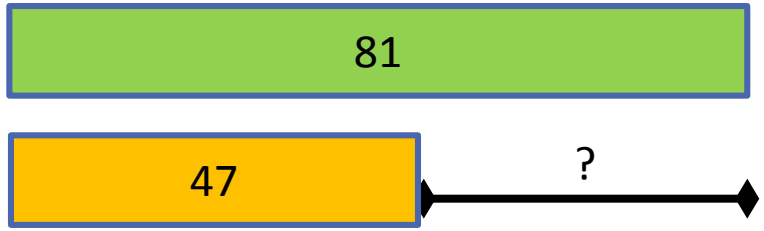
Lesson 11: Compare Situations: Pocket Survey

Lesson 12: Part-Whole and Compare Situations: Summary and Varied Practice

Compare Situations

Tia and Frances are planting a garden. They start by planting carrots and peppers. There are 47 carrot seeds and 81 pepper seeds. How many more pepper seeds did they plant than carrot seeds?

Model this situation.



Are there more carrot or more pepper seeds?

More pepper seeds.

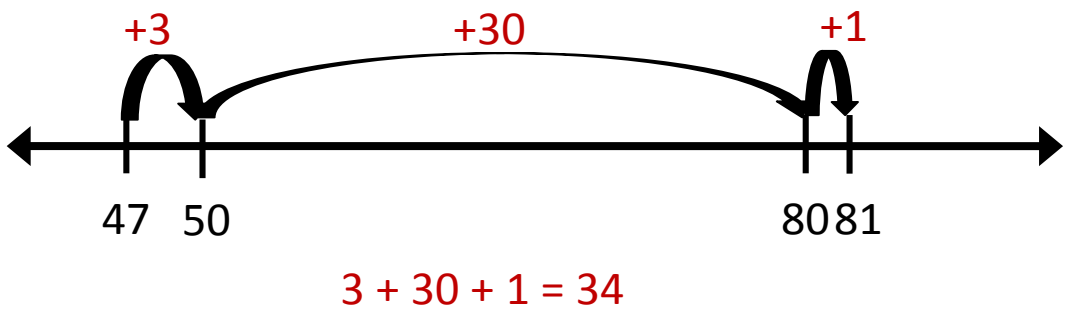
Write a number sentence for this situation. *47 + ? = 81* *81 – 47 = ?*

Now, answer the question. *34*

Compare Situations

Tia and Frances are planting a garden. They start by planting carrots and peppers. There are 47 carrot seeds and 81 pepper seeds. How many more pepper seeds did they plant than carrot seeds?

Let's examine three ways Tia solved the problem.

Number Line	Place Value	Compose with Friendly Numbers
 3 + 30 + 1 = 34	81 – 47 = ? 81 – **40** = 41 41 – **1** = 40 40 – **6** = **34**	47 + **3** = 50 50 + **30** = 80 80 + **1** = 81 So, 3 + 30 + 1 = **34**

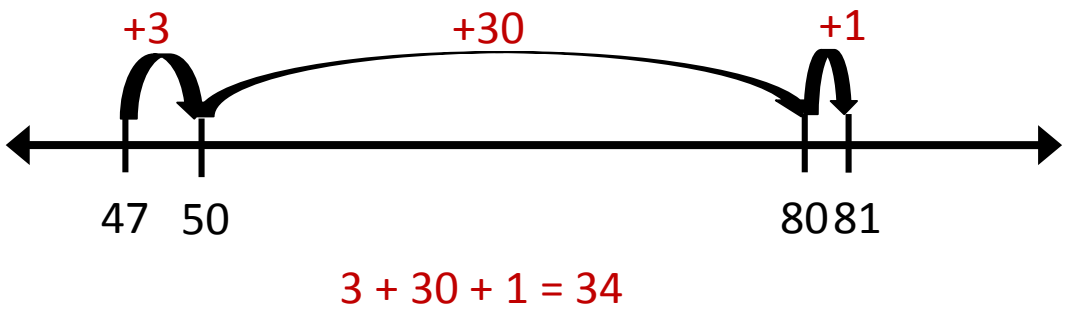
Compare Situations

Explain how each of Tia's models work using the word bank to the right.

Word Bank

Unit
Decompose
Compose
Equation

Number Line



3 + 30 + 1 = 34

In the number line model, Tia added 3 to 47 to get 50. Then she added 30 to get to 80. Then she added 1 to get to 81. She then composed 3 + 30 + 1 to get 34.

There are 34 more green been seeds than carrot seeds.

Compare Situations: Practice

Use the compare problem worksheet to solve the following problems. Model each situation first, write an equation and then solve it using one of the methods listed.

Problems	Number Sets
1. Tia planted 26 green peppers and 46 carrots. How many more carrots did she plant than green peppers?	(38, 47) (125, 75) (184, 107)
2. Frances planted 75 green peppers and 55 carrots. How many more green peppers did he plant than carrots?	(84, 15) (134, 54) (163, 89)
3. Tia planted 65 green peppers. She planted 35 more carrots than green peppers. How many carrots did she plant?	(80, 25) (72, 29) (102, 17)
4. Frances planted 70 green peppers. He planted 52 fewer carrots than green peppers. How many carrots did he plant?	(143, 52) (185, 90) (162, 34)

Compare Problem Worksheet

Story Problem	
Bar Model to represent the story.	
Equation to represent the story	
Solve - **Number line** - **Place value** - **Compose friendly number**	

Compare Situations: Practice Extension

Word Bank

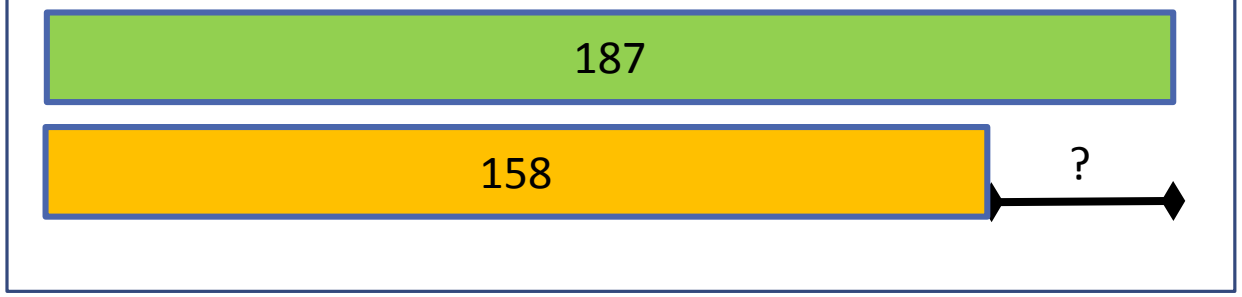
Unit
Decompose
Compose
Equation

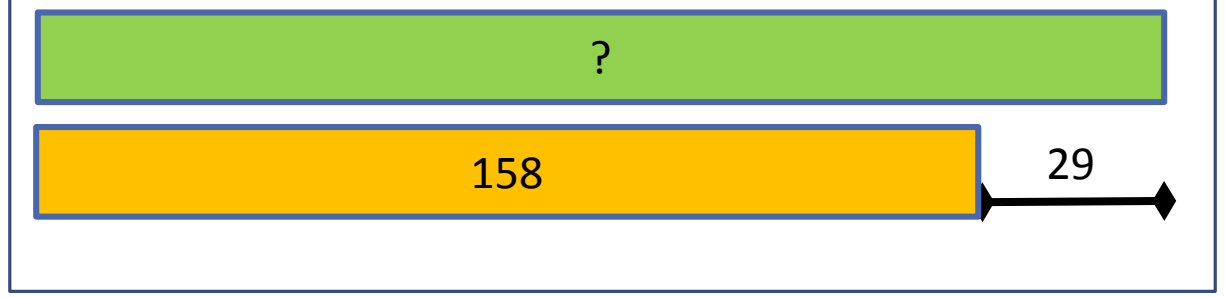
Using the word bank, explain how you solved each of the four problems.

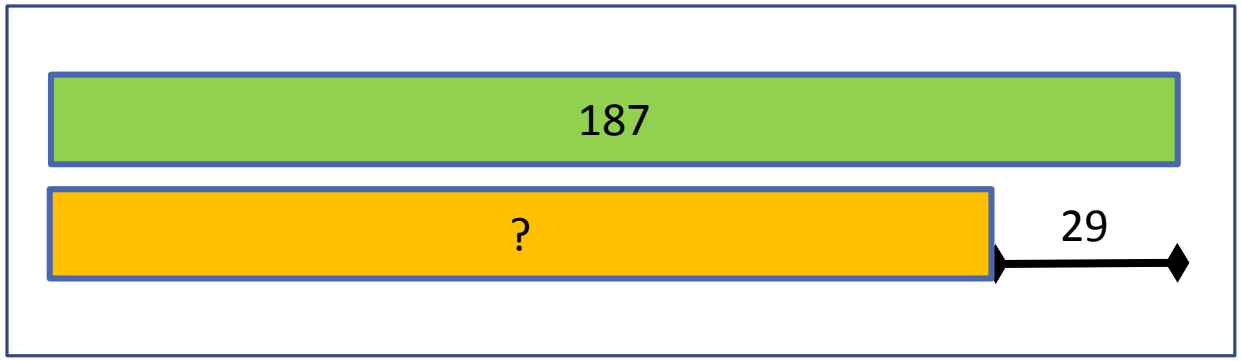
Problems	Explanation
1. Tia planted 26 green peppers and 46 carrots. How many more carrots did she plant than green peppers?	
2. Frances planted 75 green peppers and 55 carrots. How many more green peppers did he plant than carrots?	
3. Tia planted 65 green peppers. She planted 35 more carrots than green peppers. How many carrots did she plant?	
4. Frances planted 70 green peppers. He planted 52 fewer carrots than green peppers. How many carrots did he plant?	

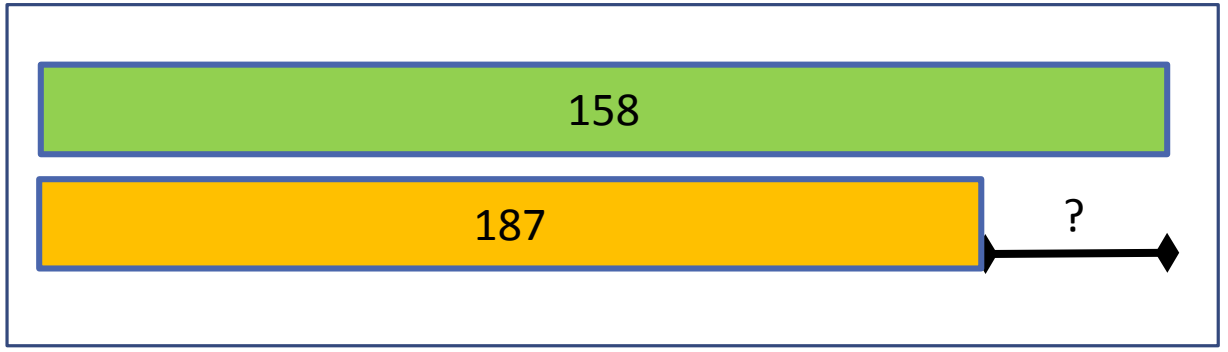
Compare Situations: Model Matching

Match the following compare models – bar model, equation and story problem.

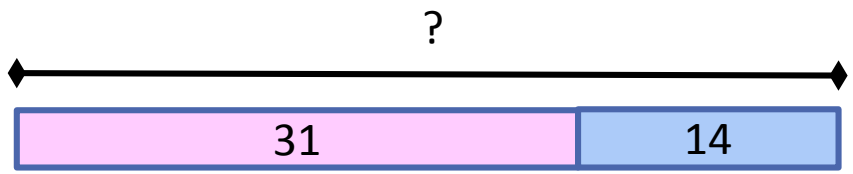
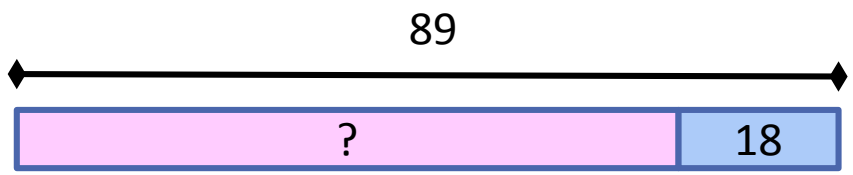




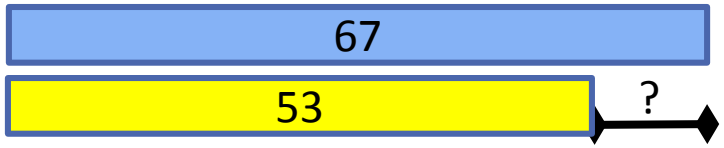




Lesson 12: Part-Whole Situations

Story Problem	Bar Model	Equation	How I solved the problem…
There were 28 children swimming in the pool. 18 of the children were girls. How many were boys?			
			
		136 + 19 = ?	
			

Lesson 12: Compare Situations

Story Problem	Bar Model	Equation	How I solved the problem…
A farmer has 38 carrots and 53 peppers. How many more peppers does the farmer have than carrots?			
			
		36 – 19 =	
A farmer has a garden with 85 animals. There are 32 more chickens than pigs. How many chickens and pigs are on the farm?			

Evidence Supporting the Modules

Idaho Standardized Achievement Test (ISAT)
Spring 2016
Grade 3 Report

The “Teacher” used the DMTI modules exclusively as her core curriculum as part of a pilot. Her grade level team did not.

Average Scale Score, Percent Proficient and Performance on Each Claim Achievement Category
Smarter Summative Mathematics Grade 3 Test for Students in [redacted]

Breakdown By: ALL Test Event: ALL GO Comparison: ON

Name	Number of Students	Average Scale Score	Percent Proficient	Claims	Claim Average Scale Score	Percent at Each Claim Achievement Category
Idaho	22942	2435 ±1	52	**Mathematics**	**2435** ±1	
				Concepts and Procedures	2437 ±1	N/A
				Problem Solving and Modeling & Data Analysis	2429 ±1	N/A
				Communicating Reasoning	2431 ±1	N/A
District	334	2434 ±4	49	**Mathematics**	**2434** ±4	
				Concepts and Procedures	2434 ±4	29 42 29
				Problem Solving and Modeling & Data Analysis	2428 ±4	23 53 24
				Communicating Reasoning	2436 ±5	15 58 26
School	103	2431 ±8	50	**Mathematics**	**2431** ±8	
				Concepts and Procedures	2433 ±9	31 36 33
				Problem Solving and Modeling & Data Analysis	2423 ±8	25 47 28
				Communicating Reasoning	2429 ±9	20 46 34
Teacher	25	2480 ±14	76	**Mathematics**	**2480** ±14	
				Concepts and Procedures	2489 ±15	8 32 60
				Problem Solving and Modeling & Data Analysis	2454 ±15	12 44 44
				Communicating Reasoning	2488 ±17	12 16 72

Developing Mathematical Thinking Institute

Professional Development

Curricular Resources

Assessment

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