

Overcoming Obstacles to Make Mathematics Work FOR Students

Matt Larson
NCTM President
@mlarson_math

Presentation is Posted at NCTM.org/larson

Goals

- Provide an overview of *Principles to Actions* – effective instructional practices and policies.
- Examine some of these concepts through the window of communication with parents.

High Quality Standards are Necessary for Effective Teaching and Learning, But Insufficient

Standards do not describe or prescribe the essential conditions required to make sure mathematics works for all students.



NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Guiding Principles for School Mathematics

1. Teaching and Learning

2. Access and Equity

3. Curriculum

4. Tools and Technology

5. Assessment

6. Professionalism

Essential Elements
of Effective Math
Programs



NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Principles to Actions: Ensuring Mathematical Success for All

The overarching message is that effective teaching is the non-negotiable core necessary to ensure that all students learn mathematics.



NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

We Must Focus on Instruction

Teaching has 6 to 10 times as much impact on achievement as all other factors combined ... Just three years of effective teaching accounts on average for an improvement of 35 to 50 percentile points.

Schmoker, M. (2006). *Results now: How we can achieve unprecedented improvements in teaching and learning*. Alexandria, VA: Association for Supervision and Curriculum Development.

Teaching and Learning Principle

Teaching and Learning. An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Eight Research-Informed Instructional Practices

- Establish mathematics **goals** to focus learning.
- Implement **tasks** that promote reasoning and problem solving.
- Use and connect mathematical **representations**.
- Facilitate meaningful mathematical **discourse**.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Eight Research-Informed Instructional Practices

- Pose purposeful **questions**.
- Build **procedural fluency** from conceptual understanding.
- Support **productive struggle** in learning mathematics.
- **Elicit and use evidence** of student thinking.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Obstacles to Implementing Research-Informed Instructional Practices

Dominant cultural beliefs about the teaching and learning of mathematics continue to be obstacles to consistent implementation of effective teaching and learning in mathematics classrooms.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Discussion Question

With a shoulder partner:

What cultural beliefs about the teaching and learning of mathematics do you believe stand as obstacles to implementation of effective teaching and learning in mathematics classrooms?

2

Beliefs about teaching and learning mathematics

Unproductive beliefs	Productive beliefs
Mathematics learning should focus on practicing procedures and memorizing basic number combinations.	Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.
Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.	All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.
Students can learn to apply mathematics only after they have mastered the basic skills.	Students can learn mathematics through exploring and solving contextual and mathematical problems.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Unproductive Belief

Students need only learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.

Eight Research-Informed Instructional Practices

Build procedural fluency from conceptual understanding.

Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding ...



NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Build procedural fluency from conceptual understanding Teacher and student actions

What are teachers doing?

Providing students with opportunities to use their own reasoning strategies and methods for solving problems.

Asking students to discuss and explain why the procedures that they are using work to solve particular problems.

Connecting student-generated strategies and methods to more efficient procedures as appropriate.

Using visual models to support students' understanding of general methods.

Providing students with opportunities for distributed practice of procedures.

What are students doing?

Making sure that they understand and can explain the mathematical basis for the procedures that they are using.

Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems.

Determining whether specific approaches generalize to a broad class of problems.

Striving to use procedures appropriately and efficiently.



NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

What are Teachers Doing?

Connecting student-generated strategies and methods to more efficient procedures.

Using visual models to support students' understanding of general methods.

The Pedagogical Value of Standard Algorithms

Standard algorithms sacrifice ease of understanding in favor of computational efficiency, and that made sense once. In today's world we have readily accessible machines to do calculations, so we can turn the educational focus on understanding the place-value system that lies beneath those algorithms.

What Employers Need

What employers need ... is people who can ask good questions, set up models, analyze results, and interpret mathematical answers. It used to be that employers needed people to calculate; they no longer need this. What we need is people to think and reason.



Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages, and innovated teaching*. San Francisco, CA: Jossey-Bass. P. 27

The Area Model Builds Understanding of the Standard Algorithm

$$\begin{array}{r}
 43 \\
 \times 17 \\
 \hline
 301 \\
 + 430 \\
 \hline
 731
 \end{array}$$

	40 + 3		
10	400	30	→ 430
+			
7	280	21	→ 301
			<u>731</u>

Moving Forward: Consider How you Communicate with Parents

We should emphasize **visual representations or models to build understanding** -- not "alternate" "different" or "new" algorithms

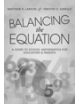
How, Why and When

$$\begin{array}{r}
 43 \\
 \times 17 \\
 \hline
 301 \\
 + 430 \\
 \hline
 731
 \end{array}$$

	40 + 3		
10	400	30	→ 430
+			
7	280	21	→ 301
			<u>731</u>

We Need to Be Clear: There is No "New Math" & There is No Such Thing as Common Core Math

At the K-8 level there is no "new math," but there are "new" research-informed instructional strategies!



Larson, M. R., & Kanold, T. D. (2016). *Balancing the equation: A guide to school mathematics for educators and parents*. Bloomington, IN: Solution Tree.


Unproductive Belief

An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.

Eight Research-Informed Instructional Practices

Support productive struggle in learning mathematics.


Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.



NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Support Productive Struggle in Learning Mathematics

Teachers sometimes perceive student frustration or lack of immediate success as indicators that they have somehow failed their students.



NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Struggle vs. Frustration

Struggle does not mean needless frustration or extreme levels of challenge. It means students expend some effort to make sense of mathematics.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning*. Charlotte, NC: Information Age Publishing

Successful Productive Struggle

- Engages students with a worthwhile task – one that captures the central idea of a lesson.
- Stretches students' thinking and performance just beyond the level they can do on their own.
- Teachers provide timely assistance.

Emerling, B., Hiebert, J., & Gallimore, R. (2015, December 7). Beyond growth mindset: Creating classroom opportunities for meaningful struggle. *Education Week Teacher*. Retrieved online at www.edweek.org/tm/articles/2015/12/07/beyond-growth-mindset-creating-classroom-opportunities-for.html

Perseverance: Learning from Our Mistakes

With parents we should talk about **perseverance and learning from mistakes**, not "productive struggle."



Larson, M. R., & Kanold, T. D. (2016). *Balancing the equation: A guide to school mathematics for educators and parents*. Bloomington, IN: Solution Tree.

If your students are going home at the end of the day less tired than you are, the division of labor in your classroom requires some attention.

William, D. (2011). *Embedded formative assessment*. Bloomington, IN: Solution Tree Press.

Eight Research-Informed Instructional Practices

Facilitate meaningful mathematical discourse.

Effective teaching of mathematics facilitates discourse among students in order to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.



NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Facilitate Meaningful Mathematical Discourse

Students who learn to articulate and justify their own mathematical ideas, reason through their own and others' mathematical explanations, and provide a rationale for their answers develop a deep understanding ...

Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary schools*. Portsmouth, NH: Heinemann.

Five Practices to Promote Productive Math Discussions

Anticipating ... Monitoring ... Selecting ...
Sequencing ... Connecting

Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions*. Reston, VA: NCTM.

Eight Research-Informed Instructional Practices

Pose purposeful questions.

Effective teaching of mathematics uses purposeful questions to assess and advance student reasoning and sense making about important mathematical ideas and relationships.

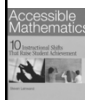
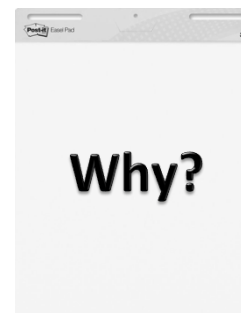
NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Effective Teachers are Effective Questioners

“Effective mathematics teachers ... pose more questions with higher cognitive demand and ask more follow-up questions”

McRel. (2010). *What we know about mathematics teaching and learning*, third edition. Bloomington, IN: Solution Tree Press.

Make “Why?” “How do you know?”
“Can you explain?”
“Do you agree/disagree?”
Classroom Mantras



Leinwand, S. (2009). *Accessible mathematics: 10 instructional shifts that raise student achievement*. Portsmouth, NH: Heinemann.

Five Essential Elements of Effective Mathematics Programs

Effective teaching and learning, while the non-negotiable core of successful mathematics programs, are part of a system of essential elements of excellent mathematics programs.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.


Five Essential Elements of Effective Mathematics Programs

Access and Equity
Curriculum
Tools and Technology
Assessment
Professionalism

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Guiding Principles for School Mathematics: Access and Equity

Access and Equity. An excellent mathematics program requires that all [each and every] students have access to high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential.




NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

NCTM Broadens its Access and Equity Work

- NCTM has re-framed its work to focus on Access, Equity, and Empowerment, to capture the critical constructs of identity, agency, and social justice.


Access and Equity Obstacles

A range of obstacles exists ... one of these involves the quality of instruction available to different groups of students ... another involves differential opportunities to learn high-quality grade level mathematics content and to be held to high expectations for mathematics achievement.



NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.


Beliefs about access and equity in mathematics, <i>continued</i>	
Unproductive beliefs	Productive beliefs
Mathematics learning is independent of students' culture, conditions, and language, and teachers do not need to consider any of these factors to be effective.	Effective mathematics instruction leverages students' culture, conditions, and language to support and enhance mathematics learning.
Tracking promotes students' achievement by allowing students to be placed in "homogeneous" classes and groups where they can make the greatest learning gains	The practice of isolating low-achieving students in low-level or slower-paced mathematics groups should be eliminated.
Only high-achieving or gifted students can reason about, make sense of, and persevere in solving challenging mathematics problems.	All students are capable of making sense of and persevering in solving challenging mathematics problems and should be expected to do so. Many more students, regardless of gender, ethnicity, and socioeconomic status, need to be given the support, confidence, and opportunities to reach much higher levels of mathematical success and interest.



NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Different Opportunities for Different Students


The learning opportunities provided for low-ability, average-ability, and high ability-grouped classrooms are hierarchically different.



Boaler, J., William, D., & Brown, M. (2000). Students' experiences of ability grouping – disaffection, polarisation and the construction of failure. *British Educational Research Journal*, 26(5), 631-648.

Educide by the Low-Slow Group

Low expectations often result in self-fulfilling prophecies. Once placed in the low tracks, it is very difficult for students to move to a higher track.



Flores, A. (2008). The opportunity gap. *TODOS Research Monograph: Promoting High Participation and Success in Mathematics by Hispanic Students: Examining Opportunities and Probing Promising Practices*, 1(1), 1-18.

Educide by the Low-Slow Group

Too often, schools serving large populations of minority students emphasize “slowing down” or providing less mathematics content, rather than providing more challenging content.

Walker, E. N. (2007). Why aren't more minorities taking advanced math? *Educational Leadership*, 65(3), 48-53.

Access Remains a Critical Issue

Across OECD countries, more than 70% of students attend schools whose principal reported that students are grouped by ability for math ... reducing ability-grouping can reduce the influence of socio-economic status on students' opportunities to learn.

OECD. (2016). *Equations and inequalities: Making mathematics accessible to all*. Paris: PISA OECD Publishing. Downloaded at <http://dx.doi.org/10.1787/9789264258495-en>.

On Paper We've De-Track High School

Even with universal algebra policies, there is the real possibility that 'tracking' will create different 'algebras' for different populations of students.

Stein, M. K., Kaufman, J. H., Sherman, M., & Hillen, A. F. (2011). Algebra: A challenge at the crossroads of policy and practice. *Review of Educational Research*, 81, 453-492.

Tracking Persists in New Forms

Although many schools have done away with traditional three-track sorting, hidden forms of tracking persist ... For example, an algebra course might sort students into fast and slow speeds of learning, so that by the end of the year students in the same class have not had the same opportunity to learn.

AERA. (2006). Do the math: Cognitive demand makes a difference. *Research Points: Essential Information for Education Policy*, 4(2).

Are the Teachers Tracked?

Teachers themselves are tracked, with those judged to be the most competent, experienced, or high status assigned to the top tracks and those with the least experience and training assigned to the lower tracks.

Darling-Hammond, L. (2007). The flat earth and education: How America's commitment to equity will determine our future. *Educational Researcher*, 36(6), 318-334.

Who is Teaching Whom?

In a study of 29 districts in 16 states, marginalized students in grades 4 through 8 had access to less effective instruction than non-marginalized students, and that lack of access persisted over time.

Isenberg, E., Max, J., Gleason, P., Potamites, L., Santillano, R., Hock, H., & Hansen, M. (2013). *Access to effective teaching for disadvantaged students* (NCEE 2014-4001). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education.

We Need to Reflect on Course and Teacher Assignment Practices

Schools should carefully study whether their teacher assignments and tracking practices are helping or hindering equity ...

Lubienski, S. T. (2007). What can we do about achievement disparities? *Educational Leadership*, 65(3), 54-59.


We expect that the very best doctors will treat the most grievously ill patients.

It should be no different in education. Great teachers have the skills to help the students who struggle the most.

Education Trust. (2005). *Gaining traction, gaining ground: How some high schools accelerate learning for struggling students*. Washington, DC: Education Trust.

Guiding Principles for School Mathematics: Curriculum

Curriculum. An excellent mathematics program includes curriculum that develops important mathematics along coherent learning progressions and develops connections among areas of mathematical study and between mathematics and the real world.

 NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.


A Curriculum Obstacle

Grade level mathematics curriculum standards are often treated as a checklist of topics. When conceptualized as such, mathematics content becomes nothing more than a set of isolated skills ...

 NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Curriculum Coherence is Critical to Support Teachers & Students

A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades (p. 14).

 NCTM. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

A Guaranteed and Viable Curriculum

What we teach – a guaranteed and viable curriculum – matters immensely. Curriculum may be the single largest factor that determines how many students in a school will learn.

Marzano, R. J. (2003). *What works in schools: Translating research into action*. Alexandria, VA: ASCD.
Schmoker, M. (2011). *Focus: Elevating the essentials to radically improve student learning*. Alexandria, VA: ASCD.

Guiding Principles for School Mathematics: Assessment

Assessment. An excellent mathematics program ensures that assessment is an integral part of instruction ... and informs feedback to students, instructional decisions, and program improvement.



NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Assessment Obstacle

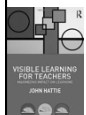
Traditionally assessment tends to emphasize the evaluation of student achievement (e.g., assigning grades), and more recently, the rating of schools and the performance of teachers – the cultural perception that links assessment to grading and rating ...



NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Who Gets the Most Valuable Feedback from Assessments?

The major reason for administering tests in classrooms is for teachers to find out what they taught well or not, who they taught well or not, and where they should focus next.



Hattie, J. (2012). *Visible learning for teachers: Maximizing impact on learning*. New York: Routledge, Taylor & Francis Group.

Valid and Reliable Classroom-Based Assessment is Not the Enemy

Assessment diagnoses student learning needs and facilitates effective instruction just as medical tests diagnose illness and direct effective treatment protocols.



Larson, M.R., & Kanold, T. K. (2016). *Balancing the equation: A guide to school mathematics for educators and parents*. Bloomington, IN: Solution Tree.

Guiding Principles for School Mathematics: Professionalism

Professionalism. In an excellent mathematics program, educators hold themselves and their colleagues accountable for the mathematical success of every student and for their personal and collective professional growth toward effective teaching and learning of mathematics.



NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Professionalism Obstacle

In too many schools, professional isolation severely undermines attempts to significantly increase professional collaboration ... A danger in isolation is that it can lead to teachers developing inconsistencies in their practice.



NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Overcoming the Obstacle: Professional Learning Communities

Teachers have a professional responsibility to participate in group decision making to improve the art and practice of teaching. One of the most powerful forums for teacher improvement is involvement in a professional learning community.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York: The Free Press.

But What Happens in Your PLCs?

A Chinese teacher sees a lesson as a performance and puts in many hours of preparation to cover the standard forty-five minute period ...

Cheng, K. (2011). Shanghai: How a big city in a developing country leaped to the head of the class. In *Surpassing Shanghai: An agenda for American education built on the world's leading systems*. Ed. M.S. Tucker. 21-50. Cambridge, MA: Harvard University Press.

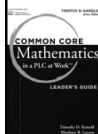
Lesson Planning is Cultural

The tendency to spend relatively little time developing lessons and to produce lesson outlines appears to be a cultural style specific to the U.S.

Ding, M., & Carlson, M. A. (2013). Elementary teachers' learning to construct high-quality mathematics lesson plans. *The Elementary School Journal*, 113(3), 359-385.

You Should Collaboratively Plan One Lesson in Each Unit

The lack of time to devote this careful planning and reflection to all lessons cannot be used as an excuse to never collaboratively learn, plan, and reflect on the effectiveness of key lessons.



Kanold, T., & Larson, M. R. (2012). *Common Core Mathematics in a PLC at Work™: Leader's Guide*. Bloomington, IN: Solution Tree Press; Reston, VA: NCTM.

Why Focus on Lesson Planning?

... the co-planning of lessons is the task that has one of the highest likelihoods of making a marked positive difference on student learning.

Hattie, J. (2012). *Visible learning for teachers: Maximizing impact on learning*. New York: Routledge, Taylor & Francis Group.

Change is Hard!

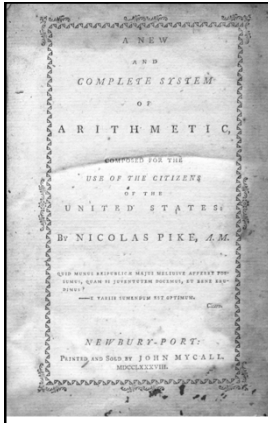
The most likely reason for the stability of teaching practices over time is that teaching is a cultural activity and cultural activities, by their very nature, are highly resistant to change.

Stigler, J. W., & Thompson, B. J. (2009). Thoughts on creating, accumulating, and utilizing shareable knowledge to improve teaching. *The Elementary School Journal*, 109(5), 442-457.

Some Practices are a Cultural Trap

Cultural routines evolve over time to enable adaptation to the environment. However, sometimes the environment changes, and yet, the cultural routine persists, even if it is now highly maladaptive.

Stigler, J. W., & Thompson, B. J. (2009). Thoughts on creating, accumulating, and utilizing shareable knowledge to improve teaching. *The Elementary School Journal*, 109(5), 442-457.



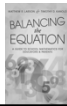
We live in the educational shadow of the 18th century.

Nicholas Pike's 1788 *Arithmetic*

Cultural Teaching Script: State a Rule, Provide an Example, Practice the Rule


Moving Forward: Support Research-Informed Instructional Practices

We expect physicians to use research-informed treatments. We must do the same.



Larson, M. R., & Kanold, T. D. (2016). *Balancing the equation: A guide to school mathematics for educators and parents*. Bloomington, IN: Solution Tree.

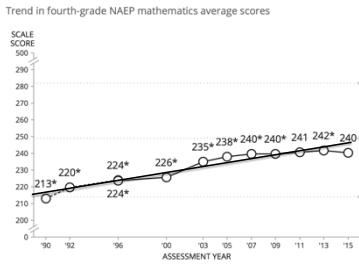
Moving Forward: Support and Implement Research-Informed Instructional Practices



The six guiding principles constitute the foundation of high-quality mathematics education.

Standards-Based Reform Has Improved Mathematics Learning

Math achievement in this country is up over the long-term ... Since we've been doing Standards-based reform!



Assessment Year	Scale Score
'90	213*
'92	220*
'96	224*
'00	224*
'03	226*
'05	235*
'07	238*
'09	240*
'11	241
'13	242*
'15	240

Standards-Based Reform Has Improved Mathematics Learning

Don't panic (yet) over the slight drop in 2015.

Subscale	Grade 4	
	2015	'15-'13
Composite	240	-1*
Number properties and operations	243	+1
Measurement	238	-1
Geometry	236	-5*
Data analysis, statistics, and probability	238	-4*
Algebra	243	-1*

* Statistically significant (p < .05).

Standards-Based Reform Has Improved Mathematics Learning

Based on the NAEP long-term trend assessment, initiated in 1973, today's fourth and eighth graders are performing at a significantly higher level than their parents and grandparents did in mathematics.

NCTM. (in press). *Mathematics education in the United States 2016: A capsule summary fact book*. Reston, VA: NCTM.

Change Takes Perseverance

"When teachers try to change more than two or three things about their teaching at the same time, the typical result is that their teaching deteriorates and they go back to doing what they were doing before."



—William, *Embedded Formative Assessment* (2011), p. 161

The Document Is Entitled *Principles to Actions*

With a shoulder partner:

- What research informed instructional strategy will you make the focus of your second semester this year?
- Who will you enlist to support you?



You Can Make It Happen!

"[Effective] teachers/leaders believe that success and failure in student learning is about what they, as teachers or leaders, did or did not do . . . We are change agents!"

—Hattie, *Visible Learning for Teachers: Maximizing Impact on Learning* (2012), p. 161

